November 2022

WORKING PAPER SERIES

2022-ACF-06

International banking regulation and Tier 1 capital ratios. On the robustness of the critical average risk weight framework

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International banking regulation and Tier 1 capital ratios. On the robustness of the critical average risk weight framework

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December 11, 2022

Abstract

Under Basel III, the new international banking regulation, banks must maintain two Tier 1 capital ratios that treat risky assets differently. The Basel Committee uses the critical average risk weight (CARW) framework, developed by the Bank of England to determine which ratio is the binding constraint. This methodology, which implicitly assumes that each asset is subject to a uniform shock, consists in comparing the implied average risk weight of a bank to a regulatory critical threshold. Using a stress test approach, we examine whether, and under which conditions, the CARW framework identifies the correct binding capital ratio. We find important errors, that are attributable to incorrect data but surprisingly not to the CARW framework. We finally generalize the methodology used by the Basel Committee and show how our stress-test approach can be used to determine which ratio is binding when only a (single class of) asset(s) is shocked.

Keywords: International banking regulation, Leverage ratio, Risk-based capital ratio, Critical average risk weight framework, Stress-test framework

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1 Introduction

In the aftermath of the financial crisis, regulators became aware of the deficiencies of the banking regulation, and in particular of the so-called risk-based capital ratio[1]. As a result, they decided to complement this risk-based capital ratio by introducing a non risk-based measure called the leverage ratio, a new Tier 1 capital ratio whose denominator is the total exposure and which incorporates both on- and off-balance sheet items. This leverage ratio is designed to act as a backstop against risk-weighted assets that would be considered as too low by regulators. According to the Basel Committee on Banking Supervision (BCBS, 2011, 2019a), it should contain the build-up of leverage in the banking sector in order to avoid deleveraging processes, as observed during the 2008 subprime financial crisis.

At the practical level, since June 2021, each bank has to comply with the Tier 1 *risk-based* capital ratio, that must be at least 6% of the risk-weighted assets, but also with a Tier 1 *non risk-based* capital ratio (the leverage ratio), that must be at least 3% of the exposure measure.[2]

At the fundamental level, as recalled in Goldstein (2017), a primary capital standard should verify the two following normative properties:

1. The regulatory capital ratio(s) should distinguish the healthy banks from the non-healthy banks.
2. The regulatory capital ratio(s) should be difficult to manipulate but easy to compute.

The risk-based capital ratio has long been criticized because it failed to satisfy the two normative properties. By allowing banks to make use of internal models to compute the risk-weighted assets, the risk-based capital ratio can be fairly easily manipulated, a practice generally called regulatory arbitrage (Barakova and Palvia, 2014, Jones, 2000, Mariathasan and Merrouche, 2014, Vallascas and Hagendorff, 2013). As a result, a high risk-based capital ratio compared to the minimum required might not reflect the creditworthiness of the bank. As a matter of fact, during the 2008 subprime financial crisis, a number of banks in difficulties had their risk-based capital ratio higher than the required minimum (see, e.g., Calomiris and Herring, 2013, Hoenig, 2012). The case of Citigroup is frequently mentioned (see, e.g., Herring, 2018). During the crisis, Citigroup reported a Tier 1 ratio greater than 11% while

[1] As noted in Paries et al. (2022), since the great financial crisis, the loss-absorbing equity buffers (i.e., capital) has been increased quantitatively and qualitatively.

[2] Note that the leverage ratio incorporates the buffer requirement for the largest banks (see BCBS, 2017). Buffers for a subset of banks should be implemented in January 2023.

its market capitalization was approximately equal to 1% of the book value of its assets, which suggests that the Tier 1 risk-based capital ratio may not always be a good indicator of the creditworthiness of the bank. This deficiency led a number of authors such as Goldstein (2017) to forcefully argue in favor of the leverage ratio.[3] According to him (see chapter 3), if Basel III could be redesigned from scratch, the capital requirement should be centered around the leverage ratio only. The risk-weights, computed under a standardized approach, would only be used as risk indicators to compute the capital surcharge. Herring (2018) (but also Greenwood et al. (2017)) makes an even more fundamental criticism of the (banking) regulation, related to its *complexity*. For instance, Herring (2018) recalls that under Basel I, a bank could compute its required capital on a piece of paper while under Basel II, its computation sometimes requires hundreds of millions of calculations due to the number of risk buckets. Herring (2018) furthers notes that in the post-crisis reform (i.e., Basel III), not only the complexity problem has not been tackled but it has even increased, in part because banks must now comply with a multitude of regulatory ratios.

The ESRB (2015) recalls that large and highly interconnected institutions have significant trading books – with low risk weights – and acknowledges that (internal) models are by essence simplifications of the real world subject to model risk, uncertainty (i.e., unknown unknowns[4]) not to mention errors, manipulation, etc. The leverage ratio, which is a simple risk-unweighted capital ratio, has been precisely introduced to limit the inherent uncertainty to assess the internal models used to compute the risk-weighted assets. When the risk-weighted assets are too low, the Tier 1 risk-based capital ratio is no longer relevant and the bank is constrained by the leverage ratio, that is, it is the binding constraint. In that sense, it acts as a simple backstop against risk weights that would be too low.

Quite interestingly, as opposed to the complexity of the internal rating-based approach, to determine the condition under which the leverage ratio will be the binding constraint,

[3]Few academic papers offer models along this idea. For instance, Acosta-Smith et al. (2020), Blum (2008) and Wu and Zhao (2016) develop simple theoretical models and show that the incentive of the bank to misreport its risk and/or the limited ability of the regulator to identify dishonest banks should argue in favour of the leverage ratio (see also Jarrow (2013) for a different argument). But the leverage ratio turns out to be also subject to a number of criticisms. For instance, in his review of the post-crisis regulatory reforms, Duffie (2017) explains that the introduction of the leverage ratio distorts the intermediation of low-risk markets such as the government securities repo because, for each unit of capital, intermediation of low risk becomes less profitable than the one of high risk. As a result, the introduction of the leverage ratio adversely impacts the liquidity of the low-risk markets. See ESRB (2015) for a review of the disadvantages (but also of the merits) of the leverage ratio.

[4]Level 3 assets are typical examples of assets for which evaluation is difficult because some parameters of the underlying model are not observable.

regulatory authorities make use of a very simple conceptual framework called the critical average risk weight (CARW) model (BoE, 2014, Fender and Lewrick, 2015) based only on aggregate quantities available in consolidated financial statements. It simply consists in comparing the implied average risk weight of a bank, defined as the total risk-weighted assets divided by the total assets, to a critical threshold approximately equal to 35%. When this implied average risk weight is lower than this critical threshold, the bank is constrained by the leverage ratio (backstop) and not anymore by the Tier 1 risk-based capital ratio.

Following BoE (2014), Fender and Lewrick (2015) and ESRB (2015, Annex 1), the Basel Committee on Banking Supervision in its regular monitoring reports makes use of the critical average risk weight (CARW) approach to determine which of the two Tier 1 ratios is the binding constraint. In 2019, they find that the fully phased-in Basel III leverage ratio is binding for 69 banks out of 166, that is, 42% (see BCBS, 2019b, section 5) while in 2020, 55 banks out of 157 are constrained by the leverage ratio[5], that is, 35% (see BCBS, 2020, section 5). This analysis of the Basel Committee on Banking Supervision however makes the two following approximations.

1. The minimum capital requirement is identical for each bank: 8.5% for the Tier 1 risk-based capital ratio (i.e., 6% from Basel III plus a 2.5% capital conservation buffer) and 3% for the leverage ratio.

2. The total value of the assets is equal to the total exposure, that is, the assets to exposure ratio is equal to one.

In practice, precisely because of Basel III (BCBS, 2019a), capital requirements are now bank-dependent in that each bank has to comply with various capital surcharges beyond the capital conservation buffer. As a result, the Tier 1 risk-based capital requirement is greater than 8.5%. In December 2018, it stood at an average of 11.75% for the larger European banks and was 11.00% for smaller institutions. In both cases, it exhibited substantial variation across institutions. Moreover, although the definition of the total exposure follows (gross) accounting values, it is not equal to the total value of the assets. In December 2018, the assets to exposure ratio stood at an average of approximately 95% for G-SIBs, G-SIIs and O-SIIs. In ESRB (2015, Annex 1), they derive the critical average risk weight when the total value of the assets is not necessarily equal to the total exposure. However, as we shall see, they fail to recognize the loss in the denominator of the capital ratios and this means that

[5]They further report that among these 55 banks, 34 come from group 1 (international active banks) while 21 come from group 2 (the rest of the banks).

their critical risk weight is correct only when the assets to exposure is equal to one. It is incorrect when this ratio is not equal to one.

It is the aim of this paper to reconsider this binding constraint problem when each bank must comply with the two Tier 1 capital ratios. As opposed to the Basel III monitoring reports (BCBS, 2019b, 2020, 2021, 2022), we make use of both the *correct model* and the *correct data* for each bank (i.e., correct capital requirement, correct asset to exposure ratio). From a theoretical point of view, to determine the binding constraint, along the line of Braouezec and Wagalath (2018) and Braouezec and Wagalath (2019), we consider a simple stress test framework in which the bank is hit by a shock (in percentage). This allows us to recognize the loss in the numerator but also in the denominator of the capital ratio, that is, we provide stress-tests foundations to the critical average risk weight model. Such a stress test formulation is very natural since in Basel III, the Tier 1 capital is explicitly designed to absorb the asset losses (i.e., asset shocks) on a going concern basis. In the simplest version of the model, the shock is a percentage of the total value of the assets but we also allow for a shock on a specific class of assets. As a by-product of this paper, we gathered a dataset of the capital requirements faced by the European banks that are subject to the regular stress-test exercises of the European Banking Authority (EBA). The data was hand-collected from the official communication of each institution for the period from December 2018 to June 2020. Equipped with the correct model (i.e., the correct average critical weight) and with the relevant database (the capital requirement of each bank at each time period), we are in a position to assess the (non) robustness of the approach followed by the Basel Committee in its monitoring reports since we are in a position to determine the correct binding constraint for each bank and each period of time.

Our results show that, compared to the case in which one both uses the correct critical average risk weight and the correct bank-dependent data, the difference is substantive. When one uses the approach followed in the Basel III monitoring reports, that is, for which the data contain two sources of inaccuracy, at least 30% of the banks are mis-classified, that is, the aggregate error regarding the binding constraint is at least 30%. However, when one considers the correct capital requirements of each bank assuming that the assets to exposure ratio is equal to one, that is, there is only one source of data inaccuracy, the aggregate error is considerably reduced and can be divided by a factor of three or four, depending upon the period. When one now considers the correct assets to exposure ratio assuming uniform capital requirements, that is, there is still only one source of data inaccuracy, the aggregate error is virtually unchanged compared to the situation in which one follows the Basel III monitoring reports. From a policy perspective, this means that the two sources of inaccuracy are far from being symmetric. Our results clearly suggest that using the correct capital requirements is by far more important than using the correct assets to exposure ratio. The Basel Committee should thus at least make use of the correct capital requirements.

It should be pointed out that we are in a position to evaluate the aggregate error when one considers *one source of inaccuracy only.* As said, in the the Basel III monitoring reports, the determination of the binding constraint contains two sources of data inaccuracy (incorrect requirements and incorrect assets to exposure ratio). However, if one wants to isolate the impact of uniform requirements on the aggregate error, considering the CARW model used by the Basel Committee with the correct assets to exposure ratio unfortunately leads to two types of inaccuracy, incorrect capital requirements and incorrect model. To perform such a study, the correct framework is needed and this is why we can offer an analysis in which only one type of inaccuracy on the aggregate error is considered. Unexpectedly, when one focuses on the aggregate error that results from the incorrect framework only (i.e., correct data but incorrect critical average risk weight, as given in ESRB (2015, Annex 1)), the error as a function of the assets to exposure ratio is invariably equal to zero. We show that when the assets to exposure ratio is lower than one – the typical case in practice – the critical average risk weight is undervalued. However, while the difference between the correct and incorrect critical average risk weight is a non-linear function of the assets to exposure ratio, it turns out to be negligible for realistic values of the parameters. This thus explains why the aggregate error is insensitive to the model error and suggests that the critical average risk weight framework is robust, at least when correct data are used.

The rest of this paper is structured as follows. In Section 2, we review the definition of the capital ratios imposed by the regulator and we detail the sample of banks used in our analysis. We examine the critical average risk weight approach used by the regulator in Section 3. We propose an alternative stress test (or shock) foundation of the critical risk weight approach in Section 4 and in section 5, we present and discuss the empirical results, descriptive statistics on the errors when using incorrect data and/or model. In Section 6, we look at the specific case of a shock affecting credit assets. Finally, we conclude in Section 7, where we also discuss some policy implications.

2 Capital ratios: Notations, definitions and basic facts

2.1 Notations and definitions

We shall adopt the following notations and definitions throughout the paper. Let $B = \{1, 2, ..., p\}$ be a set of $p \geq 2$ banks and $\mathbb{T} = \{t_1, t_2, ..., t_q\}$ be a set of $q \geq 2$ periods of time. Consider a given bank $i \in B$ at a given time $t \in \mathbb{T}$. For notational simplicity, throughout the paper, when there is no confusion, we shall drop both the bank and time subscript $(i,t) \in B \times \mathbb{T}$ but it should be clear that most quantities are both bank- and time-dependent.

Let K_1 be the Tier 1 capital of the bank, RWA be the (total) risk-weighted assets and E be the total exposure of the bank. The two Tier 1 risk-based capital ratios, denoted θ and λ respectively, are defined by regulators (e.g., BCBS, 2019a) as

$$\theta = \frac{K_1}{\text{RWA}} \quad \text{and} \quad \lambda = \frac{K_1}{E} \tag{1}$$

The total exposure E is defined as the sum of on- but also off-balance sheet exposures and thus is in theory greater than A, the total value of the assets defined as

$$A := \sum_{j \in J} A_j + m = A_{\text{risky}} + m \tag{2}$$

where A_j is the value of the risky asset $j \in J := \{1, 2, ..., n\}$ and m is the value of the cash at a given point in time.

To now consider the RWA, the total risk-weighted assets of the bank, let $\alpha_j > 0$ be the risk weight of the asset j. The total risk-weighted assets denoted RWA is frequently expressed as a weighted average of the value of each asset A_j, that is

$$\text{RWA} = \sum_{j \in J} \alpha_j A_j \tag{3}$$

and this explains the name of risk-weighted assets, that is, a weighted average of the assets.[6] Since cash (i.e., amount due from a central bank) is considered as a riskless asset, its risk weight is equal to zero. From the knowledge of $A_{\text{risky}} = A - m$ and RWA disclosed in the annual report of the bank, one can imply an *average risk weight* denoted γ defined as $\text{RWA} = \gamma \times A_{\text{risky}}$ so that

$$\gamma = \frac{\text{RWA}}{A - m} = \sum_{j \in J} \alpha_j \times \left(\frac{A_j}{A - m} \right) \tag{4}$$

From equation (4), it is now easy to see that the risk-based capital ratio as defined in equation (1) can be expressed as

$$\theta = \frac{K_1}{\gamma \times (A - m)} \tag{5}$$

[6] In the annual reports of banks, the total risk-weighted assets (RWA) are disclosed together with specific risk-weighted assets such as the credit RWA, the market RWA and the operational RWA. However, the weights α_j of each asset j is in general not explicitly disclosed.

The greater the value of m everything else equal, the higher the capital ratio. Similarly the higher the risk weights α_j, the lower the capital ratio.

Let θ_{min} and λ_{min} denote respectively the minimum regulatory Tier 1 risk-based capital ratio (RBC) requirement and the minimum regulatory leverage ratio (LR) requirement. To comply with the banking regulation, the bank must satisfy *at all times* the capital constraints, that is, it must thus be the case that

$$\theta := \frac{K_1}{\text{RWA}} \geq \theta_{min} \qquad \text{(RBC constraint)} \tag{6}$$

$$\lambda := \frac{K_1}{E} \geq \lambda_{min} \qquad \text{(LR constraint)} \tag{7}$$

2.2 Data

In this paper, we examine the subset of European banks that are subject to the stress test exercises organised periodically by the European Banking Authority (EBA). A total of 48 institutions from 15 countries participate in the exercise.[7] We report the list of those banks in Tables 1 and 2 along with their characteristics. In particular, Table 1 shows the largest European financial institutions, measured in terms of leverage ratio exposure. We follow the Financial Stability Board (FSB) and we further distinguish the ten global systemically important banks (G-SIBs)[8] from the remaining 23 global systemically important institutions (G-SIIs). Similarly, Table 2 lists the 14 smaller financial institutions identified as other systemically important institutions (O-SIIs), under the EBA classification. We study the period from December 2018 to June 2020, at a semi-annual frequency, which matches the highest reporting frequency available for some of the banks in our sample. Our sample starts in 2018, which is the year when European banks started reporting their consolidated financial statements under the IFRS 9 standards.[9] Our sample ends in June 2020, which corresponds to the period of the global outbreak of the Covid-19 pandemic that led regulators to adopt a series of temporary adjustments in the implementation of the capital requirements. We rely on the accounting data provided in the EBA data transparency exercises over our sample period. As of December 2018, while the G-SIBs had an average leverage ratio exposure of 1435.95 billion EUR, the average exposure of the G-SIIs was substantially lower, at a level

[7] We nevertheless exclude NRW Bank from our sample due to a lack of data for this bank.

[8] Although Groupe BPCE was identified as a G-SIB only in November 2019, we consider it as a G-SIB throughout our entire analysis, for comparability purposes.

[9] Banks were, prior to this period, subject to IAS 39 standards, which complicates comparisons with periods prior to 2018.

of 422.00 billion EUR. It only stands at 119.12 billion EUR for the O-SIIs. The largest G-SIB, both in terms of leverage ratio exposure and total assets is HSBC Holdings from the United Kingdom (the smallest is UniCredit from Italy). Correspondingly, the largest G-SII is Lloyds Banking Group from the United Kingdom (the smallest is Nykredit Realkredit from Denmark) and the largest O-SII is Landesbank Hessen-Thüringen Girozentrale from Germany (the smallest is Bank Polska Kasa Opieki from Poland). The Tables also report the risk weighted assets for those institutions in December 2018 (468.88 billion EUR on average for G-SIBs, 139.90 billion EUR for G-SIIs and 45.97 billion EUR for O-SIIs), their Tier 1 capital (70.92 billion EUR on average for G-SIBs, 23.11 billion EUR for G-SIIs and 7.39 billion EUR for O-SIIs) as well as the cash that they hold on their balance sheet (118.59 billion EUR on average for G-SIBs, 36.77 billion EUR for G-SIIs and 6.85 billion EUR for O-SIIs).

Table 1: Large European Financial Institutions. This Table reports the list and the characteristics of the large European banks. The reported period is December 2018. The G-SIB denomination is from the Financial Stability Board. * classified as G-SIB since 22 November 2019. Leverage ratio exposure (E), total assets (A), risk-weighted assets (RWA), Tier 1 capital (K_1) and cash (m) are in billion EUR. Tier 1 capital (K_1) and all computations based on it is phased in. A/E is multiplied by 100.

Bank Name	Country	Leverage Ratio Exposure E	Total Assets A	Assets to Exposure A/E	Risk-Weighted Assets RWA	Tier 1 Capital K_1	Cash m
Global Systemically Important Banks – G-SIBs							
HSBC Holdings Plc	United Kingdom	2283.73	2155.93	94.40	755.74	128.51	142.33
BNP Paribas	France	1864.79	1825.55	97.90	647.00	85.38	192.70
Groupe Crédit Agricole	France	1617.22	1512.09	93.50	542.09	88.51	77.69
Banco Santander S.A.	Spain	1489.09	1445.91	97.10	592.32	77.72	113.64
Barclays Plc	United Kingdom	1278.66	1264.44	98.89	348.40	59.25	198.63
Deustche Bank AG	Germany	1272.93	1348.47	105.94	350.43	55.09	197.20
Société Générale S.A.	France	1207.97	1174.87	97.26	376.05	50.81	96.58
Groupe BPCE*	France	1183.41	1168.91	98.78	392.42	62.52	83.62
ING Groep N.V.	Netherlands	1164.28	887.03	76.19	314.15	50.88	52.20
UniCredit S.p.A.	Italy	997.44	854.37	85.66	370.18	50.49	31.35
Global Systemically Important Institutions – G-SIIs							
Lloyds Banking Group Plc	United Kingdom	796.04	748.89	94.08	230.70	41.96	61.60
Group Crédit Mutuel	France	738.31	700.72	94.91	272.04	48.70	68.04
The Royal Bank of Scotland Group Plc	United Kingdom	720.61	782.53	108.59	210.94	40.49	114.53
Banco Bilbao Vizcaya Argentaria S.A.	Spain	705.30	657.12	93.17	348.26	45.95	58.30
Intesa Sanpaolo S.p.A.	Italy	668.56	639.07	95.59	276.45	42.10	56.74
Coöperatieve Rabobank U.A.	Netherlands	607.85	590.44	97.14	200.53	39.07	74.43
Nordea Bank Group	Finland	528.16	514.80	97.47	155.89	26.98	47.56
Commerzbank AG	Germany	486.34	462.24	95.05	180.50	24.11	53.91
ABN AMRO Group N.V.	Netherlands	481.42	381.62	79.27	105.39	20.29	36.26
DZ Bank AG	Germany	441.67	425.30	96.29	132.15	19.852	60.58
Danske Bank	Denmark	439.06	416.25	94.81	100.18	20.16	11.60
CaixaBank S.A.	Spain	344.90	335.65	97.32	146.15	19.45	19.12
Svenska Handelsbanken Group	Sweden	293.53	277.69	94.61	69.00	12.82	31.18
Skandinaviska Enskilda Banken Group	Sweden	270.47	222.27	82.18	69.87	13.76	23.76
KBC Group NV	Belgium	266.58	252.27	94.63	94.87	16.14	18.80
La Banque Postale	France	263.00	242.42	92.17	69.89	8.95	2.26
Landesbank Baden-Württemberg	Germany	260.12	238.22	91.58	80.35	13.04	25.78
DNB Bank Group	Norway	256.64	237.64	92.60	105.66	18.97	12.98
Erste Group Bank AG	Austria	250.19	236.26	94.43	114.60	16.52	17.54
Bayerische Landesbank	Germany	241.12	220.24	91.34	65.59	9.97	6.84
Banco de Sabadell S.A.	Spain	221.80	222.26	100.21	80.34	10.78	23.49
Swedbank Group	Sweden	218.59	201.65	92.25	62.20	11.19	17.21
Nykredit Realkredit	Denmark	205.76	193.89	94.23	46.17	10.23	3.25
Descriptive Statistics							
Mean		729.26	692.03	94.23	239.59	37.60	61.57
G-SIBs		1435.95	1363.76	94.56	468.88	70.92	118.59
G-SIIs		422.00	399.98	94.08	139.90	23.11	36.77
Standard Deviation		541.99	523.59	6.34	186.50	28.08	55.32
G-SIBs		390.37	400.00	8.22	153.23	24.86	61.52
G-SIIs		198.08	197.57	5.54	81.85	13.04	27.96

Table 2: Smaller European Financial Institutions. This Table reports the list and the characteristics of the smaller European banks. The reported period is December 2018. Leverage ratio exposure (E), total assets (A), risk-weighted assets (RWA), Tier 1 capital (K_1) and cash (m) are in billion EUR. Tier 1 capital (K_1) and all computations based on it is phased in. A/E is multiplied by 100.

Bank Name	Country	Leverage Ratio Exposure E	Total Assets A	Assets to Exposure A/E	Risk-Weighted Assets RWA	Tier 1 Capital K_1	Cash m
Other Systemically Important Institutions – O-SIIs							
Landesbank Hessen-Thüringen Girozentrale	Germany	174.61	162.12	92.85	54.28	8.88	7.20
Banco BPM S.p.A.	Italy	172.52	160.34	92.94	64.32	7.89	2.25
Norddeutsche Landesbank Girozentrale	Germany	167.13	155.88	93.27	44.89	3.38	1.50
Raiffeisen Bank International AG	Austria	163.08	139.85	85.76	72.67	10.93	22.56
Belfius Banque SA	Belgium	148.31	146.77	98.96	52.07	8.83	8.41
Unione di Banche Italiane S.p.A.	Italy	130.93	123.33	94.20	61.04	7.14	9.68
OP Financial Group	Finland	125.60	121.48	96.72	52.07	10.76	12.37
N.V. Bank Nederlandse Gemeenten	Netherlands	122.23	137.51	112.50	12.10	4.61	1.59
Bank of Ireland Group plc	Ireland	109.39	107.60	98.36	47.77	7.65	6.18
Allied Irish Banks Group plc	Ireland	94.09	91.54	97.29	51.60	11.14	7.37
Jyske Bank	Denmark	85.53	80.34	93.94	25.23	4.55	1.97
Powszechna Kasa Oszczednosci Bank Polski SA	Poland	78.30	74.89	95.65	46.60	8.17	6.77
OTP Bank Nyrt.	Hungary	48.45	45.59	94.09	29.89	4.93	4.82
Bank Polska Kasa Opieki SA	Poland	47.51	44.44	93.55	29.10	4.60	3.27
Descriptive Statistics							
Mean		119.12	113.69	95.72	45.97	7.39	6.85
Standard Deviation		43.72	40.55	5.80	16.59	2.62	5.59

From Basel III (see, e.g., BCBS, 2019a), the minimum requirements for the Tier 1 risk-based capital (θ_{min}) and leverage (λ_{min}) ratios are

$$\theta_{min} = 6\% \text{ and } \lambda_{min} = 3\% \tag{8}$$

However, since a few years, the minimum risk-based capital requirement is now *bank-dependent* (and also *time-dependent*), which means that θ_{min} depends upon the characteristics of the bank. For instance, the G-SIBs are subject to specific capital surcharges in the form of a systemic risk buffer which depends upon 12 indicators (summarized in five). In addition, a bank may also have to comply with the capital conservation buffer and/or the countercyclical buffer. Moreover, since the banking union, the European G-SIBs supervised by the European Central Bank (ECB) also have to comply with the SREP Pillar 2 (the so-called as P2R) requirement, which constitutes an additional capital surcharge. Similar requirements are also in place in other jurisdictions. For instance, in the United Kingdom, the Prudential Regulation Authority (PRA) imposes a similar Pillar 2 requirement. Overall, the minimum Tier 1 capital ratio may consequently be well above the minimum requirement of 6%. In the same vein, the leverage ratio requirement for the largest banks will be augmented with specific buffers in the near future.[10] Due to the absence of a centralised dataset, we collected manually the capital requirements for each bank and for each period of our sample. In most cases, the bank disclosed this information in its annual reports or in its official communication to investors. In Tables 3 (G-SIBs and G-SIIs) and 4 (O-SIIs), we report the capital requirements for our sample of financial institutions, and we compare them to the effective capital ratios maintained by those banks. While the average risk-based capital requirement of the largest financial institutions stands at 11.75% (11.10% for G-SIBs and 12.04% for G-SIIs), the banks have maintained an average ratio of 16.32% (15.17% for G-SIBs and 16.83% for G-SIIs), which is well in excess of the requirements. This holds in the case of the O-SIIs, which maintained an average of 17.44%, above the average requirement of 11.00%. In Tables 20 (G-SIBs and G-SIIs) and 21 (O-SIIs) in the Appendix, we further provide the decomposition of the risk-based capital requirements into their individual components. Specifically, we report the requirements in terms of common equity Tier 1 (CET1) and additional Tier 1 (AT1) under pillars 1 and 2, as well as the capital conservation buffer (CCoB), the systemic risk buffer (SyRB) and the counter-cyclical buffer (CCyB). Their sum gives the risk-based capital requirements reported in Tables 3 and 4. In Behn et al. (2022b), as one may expect, they show that systemic banks reduce a range of activities at year-end, leading to lower G-SIBs buffers[11]. Regarding the leverage ratio requirement for December 2018, it averaged 3.65% for G-SIBs, 3.19% for G-SIIs and 3.00% for O-SIIs. The effective leverage ratio that banks maintained in this period was 4.88% for G-SIBs, 5.34% for G-SIIs and 6.90% for O-SIIs, all above the corresponding requirements.

[10]While the implementation was initially planned for January 2022, it was delayed to January 2023 after the outbreak of the Covid-19 pandemic.

[11]See Behn et al. (2022a) for a related idea, related to the limits of model-based regulation.

Table 3: Capital Ratios for Large European Financial Institutions. The reported period is December 2018. θ (resp. θ_{min}) is the reported (resp. required) risk-based capital ratio. λ (resp. λ_{min}) is the reported (resp. required) leverage ratio. All percentages are multiplied by 100.

	Risk-Based Capital Ratio		Leverage Ratio	
Bank Name	Reported θ	Required θ_{min}	Reported λ	Required λ_{min}
Global Systemically Important Banks – G-SIBs				
HSBC Holdings Plc	17.00	12.04	5.63	3.95
BNP Paribas	13.20	10.70	4.58	3.75
Groupe Crédit Agricole	16.33	10.15	5.47	3.50
Banco Santander S.A.	13.12	10.25	5.22	3.50
Barclays Plc	17.01	12.68	4.63	3.84
Deutsche Bank AG	15.72	12.20	4.33	4.00
Société Générale S.A.	13.51	10.24	4.21	3.50
Groupe BPCE	15.93	10.15	5.28	3.50
ING Groep N.V.	16.20	11.94	4.37	3.50
UniCredit S.p.A.	13.64	10.69	5.06	3.50
Global Systemically Important Institutions – G-SIIs				
Lloyds Banking Group Plc	18.19	12.24	5.27	3.55
Group Crédit Mutuel	17.90	9.76	6.60	3.00
The Royal Bank of Scotland Group Plc	19.20	12.18	5.62	3.85
Banco Bilbao Vizcaya Argentaria S.A.	13.19	9.95	6.51	3.00
Intesa Sanpaolo S.p.A.	15.23	9.63	6.30	3.00
Coöperatieve Rabobank U.A.	19.48	11.91	6.43	3.00
Nordea Bank Group	17.31	13.00	5.11	3.00
Commerzbank AG	13.36	11.23	4.96	3.00
ABN AMRO Group N.V.	19.25	11.95	4.21	3.00
DZ Bank AG	15.02	10.34	4.49	3.00
Danske Bank	20.12	14.68	4.59	3.00
CaixaBank S.A.	13.31	9.57	5.64	3.00
Svenska Handelsbanken Group	18.58	16.83	4.37	3.00
Skandinaviska Enskilda Banken Group	19.69	16.53	5.09	3.00
KBC Group NV	17.02	11.28	6.06	3.00
La Banque Postale	12.81	9.81	3.40	3.00
Landesbank Baden-Württemberg	16.23	10.30	5.01	3.00
DNB Bank Group	17.95	16.95	7.39	6.00
Erste Group Bank AG	14.41	10.94	6.60	3.00
Bayerische Landesbank	15.21	10.54	4.14	3.00
Banco de Sabadell S.A.	13.42	9.95	4.86	3.00
Swedbank Group	17.99	16.30	5.12	3.00
Nykredit Realkredit	22.16	11.09	4.97	3.00
Descriptive Statistics				
Mean	16.32	11.75	5.20	3.33
G-SIBs	15.17	11.10	4.88	3.65
G-SIIs	16.83	12.04	5.34	3.19
Standard Deviation	2.48	2.17	0.88	0.59
G-SIBs	1.61	0.99	0.51	0.21
G-SIIs	2.65	2.48	0.98	0.65

Table 4: Capital Ratios for Smaller European Financial Institutions. The reported period is December 2018. θ (resp. θ_{min}) is the reported (resp. required) risk-based capital ratio. λ (resp. λ_{min}) is the reported (resp. required) leverage ratio. All percentages are multiplied by 100.

	Risk-Based Capital Ratio		Leverage Ratio	
Bank Name	Reported θ	Required θ_{min}	Reported λ	Required λ_{min}
Other Systemically Important Institutions – O-SIIs				
Landesbank Hessen-Thüringen Girozentrale	16.36	10.38	5.09	3.00
Banco BPM S.p.A.	12.26	10.38	4.57	3.00
Norddeutsche Landesbank Girozentrale	7.53	11.10	2.02	3.00
Raiffeisen Bank International AG	15.04	11.38	6.70	3.00
Belfius Banque SA	16.95	11.70	5.95	3.00
Unione di Banche Italiane S.p.A.	11.70	10.13	5.45	3.00
OP Financial Group	20.66	12.25	8.56	3.00
N.V. Bank Nederlandse Gemeenten	38.14	10.45	3.77	3.00
Bank of Ireland Group plc	16.01	10.43	6.99	3.00
Allied Irish Banks Group plc	21.60	11.23	11.84	3.00
Jyske Bank	18.04	11.21	5.32	3.00
Powszechna Kasa Oszczednosci Bank Polski SA	17.54	12.20	10.44	3.00
OTP Bank Nyrt.	16.48	9.83	10.17	3.00
Bank Polska Kasa Opieki SA	15.81	11.38	9.69	3.00
Descriptive Statistics				
Mean	17.44	11.00	6.90	3.00
Standard Deviation	6.95	0.76	2.86	0.00

3 The critical average risk weight framework considered by regulatory authorities

To the best of our knowledge, the critical average risk weight approach (CARW) was couched for the first time by the Financial Policy Committee in a document as of 2014 (see BoE, 2014) and was further developed in a document from the European Systemic Risk Board (see ESRB, 2015, Annex 1). This framework is now used in the regular Basel III monitoring reports of the Basel Committee to determine the binding constraint, see for instance BCBS (2019b), BCBS (2020), BCBS (2021), BCBS (2022).

3.1 Basic framework

The basic framework is the case in which the total exposures is assumed to be equal to the total assets, that is, $A = E$. This framework, also called the *critical average risk weight*, offers a simple mathematical framework to assess the Tier 1 capital ratio that will be the binding constraint, that is, the Tier 1 risk-based capital ratio (equation (6)) or the leverage ratio (equation (7)). As the name suggests, it leads to the determination of a critical threshold called the critical average risk weight. To determine the capital ratio which will be the binding constraint, the Basel Committee implicitly assumes that there is no cash, that is, $m = 0$ so that equation (4) reduces to

$$\gamma = \frac{\text{RWA}}{A} \tag{9}$$

It is usual in the literature to call the implied average risk weight γ the RWA density. Following ESRB (2015), let $K^L_{1,min}$ be the minimum Tier 1 capital charge (in currency) implied by the leverage requirement λ_{min} and let $K^R_{1,min}$ be the minimum Tier 1 capital charge (in currency) implied by the Tier 1 risk-based capital requirement θ_{min}. The two minimum capital requirements (in currency) $K^L_{1,min}$ and $K^R_{1,min}$ respectively solve

$$\frac{K^L_{1,min}}{E} = \lambda_{min} \quad \text{and} \quad \frac{K^R_{1,min}}{\gamma A} = \theta_{min}$$

Since $A = E$ by assumption, it thus follows that

$$K^L_{1,min} = \lambda_{min} A \tag{10}$$

$$K^R_{1,min} = \theta_{min} \gamma A \tag{11}$$

In ESRB (2015), $K^L_{1,min}$ and $K^R_{1,min}$ are called the capital charge implied by the leverage ratio and by the risk-based capital ratio respectively. According to ESRB (2015), when $K^L_{1,min} > K^R_{1,min}$, the leverage ratio requires *more capital* than the risk-based ratio and should thus be the *binding constraint*. In the opposite case, when $K^L_{1,min} < K^R_{1,min}$, the risk-based capital ratio should be the binding constraint. In the next section, we shall show that the determination of the binding constraint with the CARW is correct only when $A = E$. Let

$$\gamma_c = \frac{\lambda_{min}}{\theta_{min}} \tag{12}$$

be the ratio of the two Tier 1 minimum (capital ratio) requirements. From equations (10) and (11), it is immediate that $K^L_{1,min} > K^R_{1,min}$ ($K^L_{1,min} < K^R_{1,min}$) is equivalent to $\gamma < \gamma_c$ ($\gamma > \gamma_c$). We formalize this simple finding as a proposition entitled the critical average risk weight framework.

Proposition 1 ***Critical average risk weight framework*** *(see BoE (2014) and ESRB (2015, Annex 1)). Assume that $A = E$. Then, the*

- *Tier 1 risk-based capital ratio is the binding constraint if*

$$\gamma > \gamma_c \tag{13}$$

- *while the leverage ratio is the binding constraint if*

$$\gamma < \gamma_c \tag{14}$$

From equations (10) and (11), $K^L_{1,min} = K^R_{1,min}$ means that the two capital ratios are equally stringent and this is equivalent to

$$\gamma = \gamma_c \tag{15}$$

where γ_c can be interpreted as a *regulatory critical threshold* since both λ_{min} and θ_{min} are determined by regulators. This threshold γ_c has been called a *critical average risk weight* and is at the foundation of the introduction of the leverage ratio. It seems important to point out that at this stage, γ_c is bank-dependent but also time-dependent. From proposition 1, a bank will be constrained by the leverage ratio (and not anymore by the risk-based capital ratio) when its implied risk weight γ is lower than γ_c, the critical average risk weight. In that sense, the leverage ratio acts as a *backstop* against an implied risk weight γ that would be too low.[12]

3.2 The binding constraint under the simplifying assumptions of the Basel Committee

In their regular monitoring reports (see BCBS (2022), section 5, for the most recent one), the Basel Committee makes a number of assumptions to determine the binding constraint.

1. Regarding the theoretical framework, they use the critical average risk weight (CARW) approach (proposition 1).

2. Regarding the data, they make the following assumptions.

[12] If γ_c is thought of as the optimal regulatory threshold, this means that if a capital surcharge (buffer) is required for the risk-based capital ratio, that is, the required risk-based ratio is equal to $\theta_{min} + \text{buffer}_\theta$, a buffer should be added to the leverage ratio such that $\frac{\lambda_{min}+\text{buffer}_\lambda}{\theta_{min}+\text{buffer}_\theta} = \gamma_c$.

(a) The total value of the assets A is equal to the total exposure E (the assets to exposure ratio is equal to one), that is,

$$\frac{A}{E} = 1 \quad \text{for each bank and for each period} \tag{16}$$

(b) The minimum capital requirements are uniform across banks (this explains the superscript u). More precisely, $\lambda^u_{min} = 3\%$ and $\theta^u_{min} = 8.5\%$, that is

$$\gamma^u_c = \frac{3\%}{8.5\%} \approx 35.29\% \quad \text{for each bank and for each period} \tag{17}$$

Under the above assumptions, a bank is constrained by the leverage ratio if its implied average risk weight is lower than 35.29% and is constrained by the risk-based capital if its implied average risk weight is higher than 35.29%. These assumptions are implicitly used to construct the graph 76 p. 84 in BCBS (2022).

Remark 1 *In their monitoring reports (see, e.g.,BCBS (2022)), the Basel Committee makes use of a threshold equal to* $\gamma'^u_c = \frac{1}{\gamma^u_c} = \frac{8.5\%}{3\%} \approx 2.83$. *Assuming for simplicity that* $\gamma'^u_c = 2.83$, *let* $\theta = 2.83 \times \lambda$ *be the diagonal line plotted in the graph 76 such that* $K^L_{1,min} = K^R_{1,min}$, *that is, the two ratios are equally stringent. It is easy to show that* $K^L_{1,min} > K^R_{1,min}$ *is equivalent to* $\theta > 2.83 \times \lambda$, *that is, a bank constrained by the leverage ratio is such that the couple* (λ, θ) *is plotted above the diagonal line.*

As an illustration, we use this approach in Figure 1 for the sample of banks examined in this paper. In line with the representations of the regulator, the vertical axis shows the risk-based capital ratio (θ) reported by the financial institutions and the reported leverage ratio (λ) is in the horizontal axis. The Figure is for December 2018. The diagonal line separates the upper region where financial institutions are constrained by the leverage ratio from the lower region where the risk-based ratio is the binding constraint. To identify the type of financial institutions, we adopt the convention of depicting large financial institution with dots (the blue dots are the G-SIBs and the red dots are the G-SIIs) and O-SIIs with green triangles. As the Figure suggests, based on the CARW approach used by the regulator, the number of banks that is constrained by the risk-based capital ratio is overall almost equal to the number of banks constrained by the leverage ratio. However, the size of the institution seems to play a role on the binding constraint. While the larger financial institutions (the G-SIBs and the G-SIIs) seem to be more constrained by the leverage ratio, the situation is reversed for smaller banks (the O-SIIs), which are mainly located in the lower region of the Figure, and are consequently constrained by the risk-based ratio.

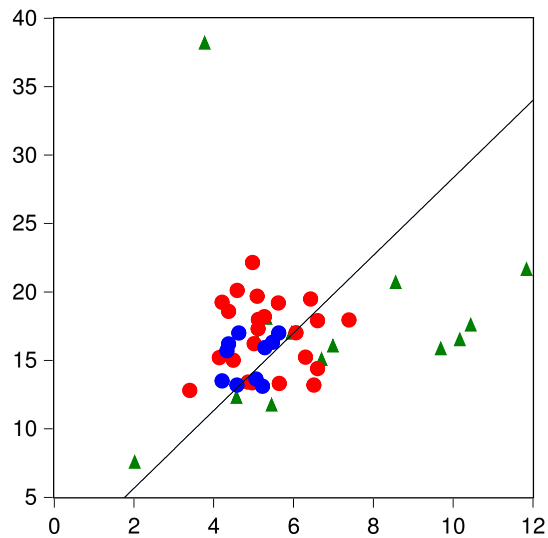


Figure 1: Uniform Capital Requirements – Monitoring Report of the Basel Committee on Banking Supervision. The reported period is December 2018. The reported risk-based capital ratio (θ) is in the vertical axis and the reported leverage ratio (λ) is in the horizontal axis. The diagonal separates the regions where the banks are constrained by the risk-based capital ratio or the leverage ratio. Its slope is 8.5%/3%. Blue (red) dots denote the G-SIBs (G-SIIs). Green triangles depict the O-SIIs.

We now look at the results for December 2018 based on proposition 1. In the spirit of the regulator, we provide a graphical representation of the results in Figure 2. The Figure shows the reported average risk weight (γ) on the vertical axis and the critical threshold (γ_c) is on the horizontal axis. For comparability purposes, we include a diagonal line with a slope of 1, that separates the upper region where banks are constrained by the risk-based capital ratio, that is, $\gamma > \gamma_c$. Correspondingly, banks located in the lower region are constrained by the leverage ratio, that is, $\gamma < \gamma_c$. We report the associated numerical values in Tables 5 and 6. We report the average risk weight (γ) as well as the critical threshold (γ_c), when all banks are subject to the same (i.e., uniform) capital requirements. We also show the corresponding binding constraint for each bank, where RBC (resp. LR) indicates that the institution is constrained by the risk-based capital (resp. leverage) ratio.

Let us make few observations. First, as Figure 2 shows, the assumption of a uniform critical threshold across financial institutions places them on a straight vertical line. There is indeed, by construction, no variability in the threshold across banks. Moreover, as Tables 1 and 2 suggest, total assets usually diverge from total exposures. In December 2018, the ratio of assets to exposure (A/E) averages 94.23% among large banks (it stands at 94.56% for G-SIBS and 94.08% for G-SIIs) and 95.72% for O-SIIs. Over that period, it ranged

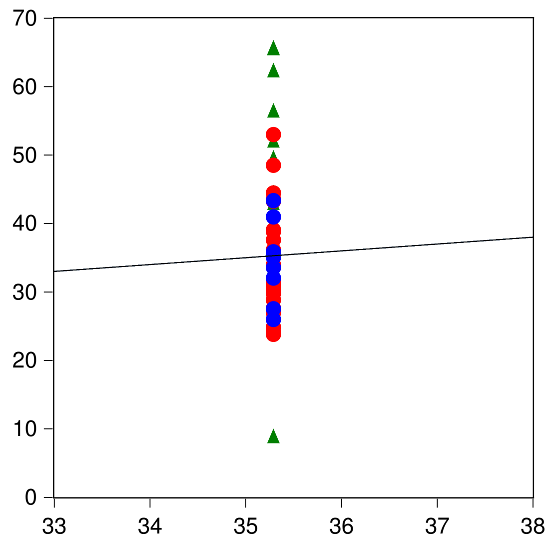


Figure 2: Critical Average Risk Weight Framework. The reported period is December 2018. γ is in the vertical axis and γ_c^u is in the horizontal axis. The Figure assumes (1) that capital requirements are uniform across banks and (2) that total assets are equal to the leverage ratio exposure ($A = E$). The diagonal separates the regions where the banks are constrained by the risk-based capital ratio or the leverage ratio. It has a slope of one. Blue (red) dots denote the G-SIBs (G-SIIs). Green triangles depict the O-SIIs.

from 112.50% for N.V. Bank Nederlandse Gemeenten to 76.19% for ING Groep, which substantially challenges the assumptions of uniform capital requirements used by the Basel Committee. In December 2018, the majority of large financial institutions were constrained by the leverage ratio according to the framework used by the Basel Committee: it is the case for 5 of the 10 G-SIBs and for 14 of the 23 G-SIIs. By contrast, 10 of the 14 O-SIIs in our sample are constrained by the risk-based capital ratio in this period.

3.3 Relaxing the simplifying assumptions made by the Basel Committee

From Tables 3 and 4, capital requirements are indeed bank-dependent. Moreover, the assets to exposure ratio is not equal to one (Tables 1 and 2). It is then natural to relax these two incorrect assumptions made in the Basel III monitoring reports by considering the actual bank-dependent quantities and to look at the consequences on the binding constraints. However, as we shall see later on, the critical average risk weight framework is correct only when the assets to exposure is equal to one. Its extension presented in ESRB (2015) annex 1 when A is not equal to E is indeed incorrect from a theoretical point of view. This means that when one considers the observed assets to exposure ratio, which is never equal to one, a model error is introduced since the framework used is not anymore valid. Let us now examine whether and how relaxing the assumption of uniform capital requirements made by the Basel Committee (i.e., we maintain the incorrect assumption that $A = E$) alters which ratio acts as the binding constraint. As we shall see, relaxing this assumption has indeed a direct impact on the results.

Looking at Figure 3, we see that financial institutions are no longer placed on a straight line but there is now, by construction, observable variability in the critical threshold (γ_c, placed in the horizontal axis of the Figure) across banks. Most institutions are placed above the diagonal, which shows that they are largely constrained by the risk-based capital ratio: it is the case for 39 banks (out of 47). We report the corresponding numerical values in Tables 5 and 6. In the last two columns, we report γ_c along with the associated binding constraint that result from the application of proposition 1 when each bank has its own (i.e., bank-dependent) set of capital requirements. The Tables and the Figure show the data for December 2018. Interestingly, while the binding constraint changes for only one G-SIB (HSBC Holdings is now constrained by the risk-based capital ratio), the application of bank-specific capital requirements alters the binding constraint of 11 G-SIIs. Whereas 9 G-SIIs were constrained by the risk-based capital ratio under the assumption of uniform capital requirements, this ratio is the binding constraint for 20 (out of 23) institutions when each bank has its own capital requirements. The results are qualitatively similar for the O-SIIs, where the leverage ratio is the binding constraint for only one bank (N.V. Bank Nederlandse Gemeenten) when bank-specific capital requirements are used, while 4 banks were constrained by this ratio under the assumption of uniform requirements. Importantly, whenever we observe a change in the binding constraint, it switches in all cases from the leverage ratio under the basic framework to the risk-based capital ratio when we relax the assumption of uniform capital requirements. This preliminary analysis suggests that relaxing the assumption of uniform capital requirements made by the Basel Committee impacts the ratio that will act as the binding constraint for European banks.

Table 5: Critical Average Risk Weight Framework for Large Banks. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based capital (resp. leverage) ratio.

		Uniform		Bank-Dependent	
Bank Name	γ	γ_c^u	Binding Constraint	γ_c	Binding Constraint
Global Systemically Important Banks – G-SIBs					
HSBC Holdings Plc	35.05	35.29	LR	32.82	RBC
BNP Paribas	35.44	35.29	RBC	35.06	RBC
Groupe Crédit Agricole	35.85	35.29	RBC	34.48	RBC
Banco Santander S.A.	40.97	35.29	RBC	34.15	RBC
Barclays Plc	27.55	35.29	LR	30.30	LR
Deutsche Bank AG	25.99	35.29	LR	32.80	LR
Société Générale S.A.	32.01	35.29	LR	34.20	LR
Groupe BPCE	33.57	35.29	LR	24.50	LR
ING Groep N.V.	35.42	35.29	RBC	29.33	RBC
UniCredit S.p.A.	43.33	35.29	RBC	32.76	RBC
Global Systemically Important Institutions – G-SIIs					
Lloyds Banking Group Plc	30.81	35.29	LR	29.00	RBC
Group Crédit Mutuel	38.82	35.29	RBC	30.75	RBC
The Royal Bank of Scotland Group Plc	26.96	35.29	LR	31.62	LR
Banco Bilbao Vizcaya Argentaria S.A.	53.00	35.29	RBC	30.16	RBC
Intesa Sanpaolo S.p.A.	43.26	35.29	RBC	31.17	RBC
Coöperatieve Rabobank U.A.	33.96	35.29	LR	25.20	RBC
Nordea Bank Group	30.28	35.29	LR	23.08	RBC
Commerzbank AG	39.05	35.29	RBC	26.71	RBC
ABN AMRO Group N.V.	27.62	35.29	LR	25.12	RBC
DZ Bank AG	31.07	35.29	LR	29.03	RBC
Danske Bank	24.07	35.29	LR	20.44	RBC
CaixaBank S.A.	43.54	35.29	RBC	31.36	RBC
Svenska Handelsbanken Group	24.85	35.29	LR	17.83	RBC
Skandinaviska Enskilda Banken Group	31.44	35.29	LR	18.15	RBC
KBC Group NV	37.61	35.29	RBC	26.61	RBC
La Banque Postale	28.83	35.29	LR	30.57	LR
Landesbank Baden-Württemberg	33.73	35.29	LR	29.14	RBC
DNB Bank Group	44.46	35.29	RBC	35.39	RBC
Erste Group Bank AG	48.51	35.29	RBC	27.44	RBC
Bayerische Landesbank	29.78	35.29	LR	28.48	RBC
Banco de Sabadell S.A.	36.14	35.29	RBC	30.14	RBC
Swedbank Group	30.85	35.29	LR	18.41	RBC
Nykredit Realkredit	23.81	35.29	LR	27.06	LR
Descriptive Statistics					
Mean	34.47	35.29		28.89	
G-SIBs	34.52	35.29		33.04	
G-SIIs	34.45	35.29		27.08	
Standard Deviation	7.16	0.00		4.91	
G-SIBs	5.28	0.00		1.90	
G-SIIs	7.95	0.00		4.73	

Table 6: Critical Average Risk Weight Framework for Smaller Banks. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based capital (resp. leverage) ratio.

		Uniform		Bank-Dependent	
Bank Name	γ	γ_c^u	Binding Constraint	γ_c	Binding Constraint
Other Systemically Important Institutions – O-SIIs					
Landesbank Hessen-Thüringen Girozentrale	33.48	35.29	LR	28.92	RBC
Banco BPM S.p.A.	40.12	35.29	RBC	28.92	RBC
Norddeutsche Landesbank Girozentrale	28.80	35.29	LR	27.02	RBC
Raiffeisen Bank International AG	51.97	35.29	RBC	26.37	RBC
Belfius Banque SA	35.48	35.29	RBC	25.65	RBC
Unione di Banche Italiane s.p.A.	49.49	35.29	RBC	29.63	RBC
OP Financial Group	42.86	35.29	RBC	24.49	RBC
N.V. Bank Nederlandse Gemeenten	8.80	35.29	LR	28.71	LR
Bank of Ireland Group plc	44.40	35.29	RBC	28.78	RBC
Allied Irish Banks Group plc	56.37	35.29	RBC	26.72	RBC
Jyske Bank	31.41	35.29	LR	26.77	RBC
Powszechna Kasa Oszczednosci Bank Polski SA	62.22	35.29	RBC	24.60	RBC
OTP Bank Nyrt.	65.57	35.29	RBC	30.53	RBC
Bank Polska Kasa Opieki SA	65.48	35.29	RBC	26.37	RBC
Descriptive Statistics					
Mean	44.03	35.29		27.39	
Standard Deviation	16.01	0.00		1.87	

4 Stress-test foundations of the critical average risk weight framework

By design, the role of Tier 1 capital is to absorb the asset losses, that is, adverse shocks, on a going concern basis (see, e.g., BCBS, 2017). In the critical average risk weight (CARW) framework, the way the binding constraint is determined does not give an explicit role to shocks but the main issue is that the CARW framework is not correct from a theoretical point of view when A is not equal to E. In this section, we offer a shock framework in which the stringency of the capital requirement is obtained by comparing two critical shocks. As a byproduct, our approach also allows us to determine the binding constraint when only a specific asset or class of assets is hit by a shock. From a theoretical point of view, our framework is similar to Braouezec and Wagalath (2018) and to Braouezec and Wagalath (2019).

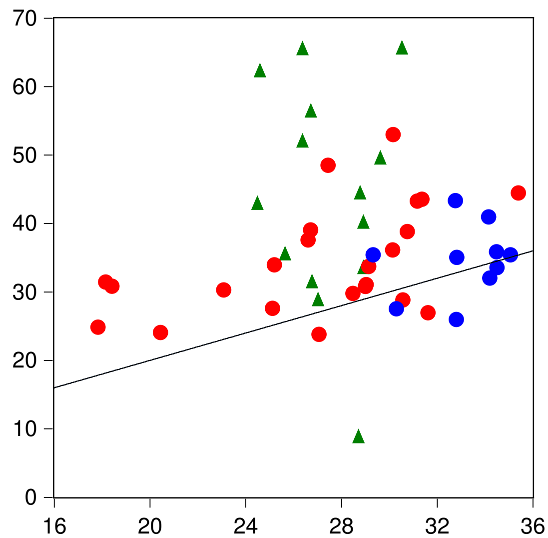


Figure 3: Critical Average Risk Weight Framework with Bank-Dependent Capital Requirements. The reported period is December 2018. γ is in the vertical axis and γ_c is in the horizontal axis. The Figure assumes that total assets are equal to the leverage ratio exposure ($A = E$). The diagonal separates the regions where the banks are constrained by the risk-based capital ratio or the leverage ratio. It has a slope of one. Blue (red) dots denote the G-SIBs (G-SIIs). Green triangles depict the O-SIIs.

4.1 Loss accounting

Let $\Delta := (\Delta_j)_{j=1}^n = (\Delta_1, \Delta_2, ..., \Delta_n) \in [0,1]^n$ be a vector of adverse shocks, where $\Delta_j \geq 0$ represents the percentage of loss on asset j. Before the shock, when $m = 0$, the total value of the assets is equal to $A = \sum_j A_j$ and the risk-weighted assets are equal to $\text{RWA} = \sum_{j \in J} \alpha_j A_j$. After a shock (loss), the value of asset j is equal to $A_j(1-\Delta_j)$ and its risk-weighted assets is equal to $\text{RWA}_j(\Delta) = \alpha_j A_j(1-\Delta_j)$. The total risk-weighted assets after the shock, denoted $\text{RWA}(\Delta)$ thus are equal to

$$\text{RWA}(\Delta) = \sum_{j \in J} \alpha_j A_j (1 - \Delta_j) = \text{RWA} - \sum_{j \in J} \alpha_j A_j \Delta_j \tag{18}$$

Note that $\text{RWA}(\Delta)$ is equal to RWA, the risk-weighted assets before the loss, minus the *risk-weighted loss* $\sum_{j \in J} \alpha_j A_j \Delta_j$, which means that the loss depends upon the risk weights. Let $\theta(\Delta)$ and $\lambda(\Delta)$ denote the risk-based and the leverage ratio after the shock.

Fact 1 *The risk-based and the leverage ratio after the shock* Δ *are given below*

$$\theta(\Delta) = \frac{K_1(\Delta)}{\text{RWA}(\Delta)} = \frac{K_1 - \sum_{j\in J} A_j\Delta_j}{\text{RWA} - \sum_{j\in J}\alpha_j A_j \Delta_j} \tag{19}$$

$$\lambda(\Delta) = \frac{K_1(\Delta)}{E(\Delta)} = \frac{K_1 - \sum_{j\in J} A_j\Delta_j}{E - \sum_{j\in J} A_j \Delta_j} \tag{20}$$

and both the numerator and the denominator are impacted by the shock Δ.

This approach in which each asset j is shocked is fairly general but turns out to be difficult to interpret from a financial point of view because it requires to compare sets rather than numbers. To see this, consider a *critical adverse shock* $\Delta \in [0,1]^n$ defined by $\theta(\Delta) = \theta_{min}$. From equation (19), this leads to $K_1 - \theta_{min}\text{RWA} = \sum_{j=1}^n A_j\Delta_j(1-\alpha_j\theta_{min})$ and this means that there are *infinitely many different shocks* $\Delta \in [0,1]^n$ that satisfy the equality $\theta(\Delta) = \theta_{min}$. Put differently, $\theta(\Delta) = \theta_{min}$ is equivalent to $\Delta \in \mathcal{S}$ where $\mathcal{S} = \{\Delta \in [0,1]^n : K_1 - \theta_{min}\text{RWA} = \sum_{j=1}^n A_j\Delta_j(1-\alpha_j\theta_{min})\}$ can be called a 'critical set'. It is not only difficult to interpret such a critical set $\mathcal{S}$ financially, but it is even more difficult to compare the critical set of two different banks. However, in the particular case in which each asset is hit by the *same adverse shock*, that is, for each asset $j = 1, 2, ..., n$, $\Delta_j = \Delta \in [0,1]$, the equality $K_1 - \theta_{min}\text{RWA} = \sum_j A_j\Delta_j(1-\alpha_j\theta_{min})$ reduces to $K_1 - \theta_{min}\text{RWA} = \sum_j A_j\Delta(1-\alpha_j\theta_{min})$, that is, there is a unique (positive) number denoted Δ such that the equality is satisfied. In what can be called the simple stress test framework, we end up with a number and not a set. Such an approach underlies the critical average risk weight framework.

4.2 A simple stress-test framework

Let $\Delta_j := \Delta \in [0,1]$ be a shock in percentage on A_j for each $j \in J$. We call such an approach the *simple stress-test* framework.

Remark 2 *This assumption of such a specific shock is, as we shall see, the implicit assumption made initially by the Bank of England and subsequently by the regulator to obtain proposition 1, called the critical average risk weight (CARW) framework.*

We shall assume that before the shock, the bank complies with the two capital ratios, that is, $\theta \geq \theta_{min}$ and $\lambda \geq \lambda_{min}$. To facilitate the comparison with the critical average risk weight framework used by the Basel Committee, we make the assumption of no cash, that is, $m = 0$. In the Appendix, we however consider the case with positive cash. Interestingly, while the value of cash is greater and greater over time, it has no impact on the results which gives an empirical reason to assume $m = 0$. From the previous discussion, we know that the value of Tier 1 capital after the shock is equal to $K_1(\Delta) = K_1 - \sum_{j \in J} A_j \Delta = K_1 - A\,\Delta$ since $\sum_{j \in J} A_j = A$. In practice, since A is much higher than K_1, we make the assumption that $A > K_1$. As a result, there exists a *lowest critical shock* $\Delta_0 < 1$ such that the bank is insolvent, that is,

$$K_1 - \Delta_0\,A = 0 \Longleftrightarrow \Delta_0 < 1 \tag{21}$$

Within our stress test framework, from equation (19), the risk-based capital ratio after the shock is equal to $\theta(\Delta) = \frac{K_1 - A\,\Delta}{\sum_{j \in J} \alpha_j A_j\,(1-\Delta)}$. Since $m = 0$, $\sum_{j \in J} \alpha_j A_j = \gamma A$, so that the risk-based capital ratio can be written as

$$\theta(\Delta) = \frac{K_1 - A\,\Delta}{\gamma A(1-\Delta)} \tag{22}$$

From equation (20), the leverage ratio right after the shock Δ is equal to

$$\lambda(\Delta) = \frac{K_1 - A\,\Delta}{E - A\Delta} \tag{23}$$

and note that both $\lambda(.)$ and $\theta(.)$ are continuous and strictly decreasing functions of $\Delta \in [0, \Delta_0]$

Let Δ_R^* be the critical shock such that the risk-based capital ratio $\theta(\Delta_R^*) = \theta_{min}$ and let Δ_L^* be the critical shock size such that $\lambda(\Delta_L^*) = \lambda_{min}$. Since both θ_{min} and λ_{min} are positive, it thus follows that both Δ_R^* and Δ_L^* are strictly lower than Δ_0. The next result is simple but fundamental since it provides a characterization of the two critical shock sizes Δ_R^* and Δ_L^* that enables us to determine the binding constraint.

Lemma 1 *Consider a given bank at a given point in time such that $\theta > \theta_{min}$ and that $\lambda > \lambda_{min}$. The critical thresholds Δ_R^* and Δ_L^* are given below.*

$$\Delta_R^* = \frac{K_1 - \theta_{min}\gamma A}{A(1-\gamma\theta_{min})} \tag{24}$$

$$\Delta_L^* = \frac{K_1 - \lambda_{min} E}{A(1-\lambda_{min})} \tag{25}$$

Proof. See the appendix.

Once the two critical thresholds Δ_R^* and Δ_L^* are known, we are in a position to determine the capital ratio which is the binding constraint. To see this, recall that the two capital ratios $\lambda(\Delta)$ are decreasing and continuous functions of $\Delta \in [0, \Delta_0)$ and assume for instance $\Delta_R^* < \Delta_L^*$. When the shock $\Delta = \Delta_R^*$, by construction, the risk-based capital ratio is equal to θ_{min}. Since $\Delta_L^* > \Delta_R^*$, this means that for a shock Δ slightly higher than Δ_R^*, say equal to $\Delta = \Delta_R^* + \epsilon$ (where ϵ is positive but can be arbitrarily small), the risk-based capital ratio is *lower* than θ_{min} while the leverage ratio is *higher* than λ_{min}. Put differently, after a shock $\Delta \in (\Delta_R^*, \Delta_L^*)$, the bank will be insolvent because it fails to comply with the risk-based capital ratio but not with the leverage ratio. It thus follows from the discussion that as long as $\Delta_R^* < \Delta_L^*$ ($\Delta_R^* > \Delta_L^*$), the binding constraint is the risk-based capital ratio (leverage ratio). The following definition shows that the binding constraint is found by comparing the two critical shocks.

Definition 1 *The Tier 1 risk-based capital ratio is said to be the binding constraint if*

$$\Delta_R^* < \Delta_L^* \tag{26}$$

while the leverage ratio is said to be the binding constraint if

$$\Delta_R^* > \Delta_L^* \tag{27}$$

The following proposition formulates the binding constraint by making use of the critical average risk weight γ_c in order to be compared with proposition 1.

Proposition 2

- *The Tier 1 risk-based capital ratio is the binding constraint if*

$$\gamma \times \underbrace{\left[\frac{A}{E}\left(\frac{1-\lambda_{min}}{1-\lambda}\right) - \left(\frac{\lambda-\lambda_{min}}{1-\lambda}\right)\right]}_{\Gamma_1\left(\frac{A}{E},\lambda_{min},\lambda\right):=\Gamma_1} > \gamma_c \tag{28}$$

- *The leverage ratio is the binding constraint if*

$$\gamma \times \left[\frac{A}{E}\left(\frac{1-\lambda_{min}}{1-\lambda}\right) - \left(\frac{\lambda-\lambda_{min}}{1-\lambda}\right)\right] < \gamma_c \tag{29}$$

Proof. *See the appendix.*

Corollary 1 *When $A = E$ in proposition 2, the binding constraint is identical to the one given in proposition 1.*

It suffices to observe that when $A = E$, the term in the bracket [...] in equation (28) reduces to one, which explains why the critical average risk weight gives a binding constraint identical to our shock framework.

4.3 On the approach followed by ESRB (2015) when the assets to exposure ratio is not equal to one

Up to now, we only considered the CARW framework when $A = E$. When $A \neq E$, the decision rule must obviously be adapted. According to ESRB (2015), using the CARW approach, it suffices to make use of E instead of A in equation (10) to obtain the extension of proposition 1 when A is not equal to E.

Proposition 3 *(ESRB (2015), annex 1).*

- *The Tier 1 risk-based capital ratio is the binding constraint if*

$$\gamma \times \left(\frac{A}{E}\right) > \gamma_c \tag{30}$$

- *The leverage ratio is the binding constraint if*

$$\gamma \times \left(\frac{A}{E}\right) < \gamma_c \tag{31}$$

Corollary 2 *When $A \neq E$, proposition 2 and 3 do not coincide so that the binding constraints may differ.*

To understand the problem with proposition 3, consider once again the methodology followed by the regulatory authorities (e.g., BoE, 2014, ESRB, 2015, BCBS, 2022). Let L^{LR}_{max} and L^{RBC}_{max} be the *maximum loss* in currency (say in EUR) for the leverage ratio and for the risk-based capital ratio respectively. According to regulatory authorities (e.g., ESRB (2015)), L^{LR}_{max} and L^{RBC}_{max} are implicitly defined such that

$$\frac{K_1 - L^{LR}_{max}}{E} = \lambda_{min} \quad \text{and} \quad \frac{K_1 - L^{RBC}_{max}}{\text{RWA}} = \theta_{min} \tag{32}$$

Using (32), it thus follows that $K_1 - L^{LR}_{max} = \lambda_{min} \times E$ and $K_1 - L^{RBC}_{max} = \theta_{min} \times \text{RWA}$ so that $K_1 - \lambda_{min}E = L^{LR}_{max}$ and $K_1 - \theta_{min}\text{RWA} = L^{RBC}_{max}$. Using the fact that $\text{RWA} = \gamma A$, it is now easy to see that $L^{RBC}_{max} < L^{LR}_{max}$ is equivalent to $\gamma \times \frac{A}{E} > \gamma_c$, which yields equation (30) of proposition 3. However, from fact 1, we know that a loss should *impact both the numerator and the denominator* of each capital ratio. Equation (32) thus is incorrect because it fails to recognize the loss in the denominator of the capital ratio, that is, neither E nor RWA recognize the loss.

This lack of recognition of the loss in the denominator thus explains why the framework followed by the Basel Committee to determine the binding constraint is incorrect from a theoretical point of view when $A \neq E$. More fundamentally, the difficulty with the approach followed by ESRB (2015) is that they implicitly use an approach in terms of a maximal loss in currency rather than in percentage. As a result, it is unclear how one should compute the risk-weighted loss and this may explain their implicit choice to recognize the loss only in the numerator of the Tier 1 capital ratios.

One can now easily obtain the critical average risk weight when A is not equal to E as a function of γ_c. Using the notations of ESRB (2015), let $\overline{\gamma}_c$ be the critical average risk weight in the CARW approach. Using proposition 3, the threshold $\overline{\gamma}_c$ is found by solving $\overline{\gamma}_c \times \frac{A}{E} = \gamma_c$ so that

$$\overline{\gamma}_c = \frac{\gamma_c}{\frac{A}{E}} \Longleftrightarrow \overline{\gamma}_c = \frac{\lambda_{min}}{\theta_{min}} \times \frac{E}{A} \tag{33}$$

Let $\widetilde{\gamma}_c$ be this critical risk weight within our shock model. Using the same logic, from proposition 2, the critical average risk weight is equal to

$$\widetilde{\gamma}_c = \frac{\gamma_c}{\left[\frac{A}{E}\left(\frac{1-\lambda_{min}}{1-\lambda}\right) - \left(\frac{\lambda-\lambda_{min}}{1-\lambda}\right)\right]} \tag{34}$$

Note that both $\widetilde{\gamma}_c$ and $\overline{\gamma}_c$ depend on $\frac{A}{E}$. To analyze the difference between these two critical average risk weights, let $x = \frac{A}{E}$. Consider now the condition under which $\widetilde{\gamma}_c$ is higher than $\overline{\gamma}_c$ and note that the denominator of equation (34) is positive as long as $x > \frac{\lambda-\lambda_{min}}{1-\lambda_{min}}$, something always true in practice. From equations (33) and (34), it is easy to see that $\widetilde{\gamma}_c > \overline{\gamma}_c$ is equivalent to $\left[x\left(\frac{1-\lambda_{min}}{1-\lambda}\right) - \left(\frac{\lambda-\lambda_{min}}{1-\lambda}\right)\right] - x < 0$, which reduces to $(\lambda - \lambda_{min})(x-1)$. As a result, when $x < 1$, $\widetilde{\gamma}_c > \overline{\gamma}_c$. The following proposition formalizes this simple finding.

Proposition 4

- *When the total exposure is higher than the total value of the assets* ($E > A$)*, the critical risk weight found in ESRB (2015) is undervalued* ($\overline{\gamma}_c < \widetilde{\gamma}_c$)*.*
- *When the total exposure is lower than the total value of the assets* ($E < A$)*, the critical risk weight found in ESRB (2015) is overvalued* ($\overline{\gamma}_c > \widetilde{\gamma}_c$)*.*

When $E > A$ (i.e., $x < 1$) as it is the case for most banks[13], the fact that $\overline{\gamma}_c$ is *undervalued* means that the prediction of the critical average risk weight framework may incorrectly predict the risk-based capital ratio to be the binding constraint while it is in reality the leverage ratio. Such a situation occurs when the observed implied risk weight $\gamma \in (\overline{\gamma}_c, \widetilde{\gamma}_c)$.

[13] As reported in Tables 1 and 2, in December 2018, 43 out of 47 banks had an assets to exposure ratio lower than one.

5 On the aggregate error due the simplifying assumptions of the Basel Committee

We display in this section tables regarding the aggregate error made in the Basel III monitoring reports (aggregate and per type of institution) but we also provide an analysis of the magnitude of each error separately. For each period of time, our database contains the relevant data, that is, the minimal capital requirements for each bank and the value of the assets to exposure ratio. Since the critical shock approach provides the correct theoretical framework (see proposition 2), using our database, we are thus in a position to determine the true binding constraint. Using proposition 1 when capital requirements and the assets to exposure ratio are uniform across banks, we can replicate the analysis of the Basel Committee in its monitoring reports and compare the binding constraints. Deviation from the correct binding constraint may in principle be attributable to the incorrect framework and/or to incorrect data. Since proposition 2 reduces to proposition 1 when A is equal to E, in the Basel III monitoring reports, errors are attributable to incorrect data only. Data contain however two types of inaccuracy, uniform capital requirement on the one hand and uniform assets to exposure ratio on the other hand. We believe that analyzing *separately* each type of inaccuracy on the aggregate error is something interesting to perform. We thus provide Tables in which we consider only one type of inaccuracy. Since we know that the critical average risk weight framework is incorrect when the assets to exposure ratio is not equal to one, we are also in a position to evaluate the aggregate error due to the inaccuracy of the model only. It is important to point out that our results consists in reporting descriptive statistics regarding the number or the percentage of banks for which the binding constraint is incorrect. To perform such an analysis, we follow a bank-by-bank approach and this means that we make no use of statistical models.

5.1 Aggregate error on the binding constraint

We determine in what follows the aggregate error that results from the two types of inaccuracy in the data.

Aggregate analysis. The overall sample contains 47 banks and our aim is to determine the aggregate error, that is, the total number of banks for which the binding constraint is incorrect. When the binding constraint is incorrect, we may simply say that the bank is incorrectly classified. To compute this aggregate error for December 2018, consider for instance the G-SIBs. In Table 5 (column uniform), the critical average risk weight framework is used to determine the binding constraint (proposition 1) assuming that $A = E$, as in the

Basel III monitoring reports, which means that the data contain two types of inaccuracy. Since Table 13 gives the correct binding constraint (i.e., it uses proposition 2 with bank-dependent requirements when A may differ from E), we can thus determine the aggregate error made in the Basel III monitoring reports. To compute the error, as said earlier, we follow a bank-by-bank approach. Consider for instance the case of the British bank HSBC, classified as a G-SIB in 2018. From Table 5 (column uniform), the binding constraint for that bank is the leverage ratio (LR). From Table 13, the correct (or true) binding constraint is the risk-based capital ratio (RBC). As a result, HSBC is incorrectly classified. We report in Table 7 the aggregate error for each period. More precisely, we report the number and the percentage of banks that are incorrectly classified.

Table 7: Overall mis-classification due to uniform requirements and $A = E$.

Errors | Periods	December 2018	June 2019	December 2019	June 2020
Perc. (abs.)	32% (15)	36.2% (17)	38.3% (18)	34% (16)

Overall, it can be seen from Table 7 that the assumptions made in the Basel III monitoring reports (i.e., inaccurate data) to determine the binding constraints yield an important error since approximately one third of banks are incorrectly classified.

Bank-specific analysis. We now look at the error when each type of institution (G-SIB, G-SII, O-SII) is considered separately.

Table 8: Mis-classification (by type of institution) due to uniform requirements and $A = E$

Errors | Periods	December 2018	June 2019	December 2019	June 2020
G-SIBs	40% (4)	40% (4)	30% (3)	0% (0)
G-SIIs	34.8% (8)	43.5% (10)	52.2% (12)	52.2% (12)
O-SIIs	21.4% (3)	21.5% (3)	21.5% (3)	28.6%(4)

As can be seen from Tables 8, the global error is far from being homogeneous. It is particularly important for G-SIIs in 2019 and 2020 as more than fifty percent of G-SIIs are incorrectly classified. Moreover, the error seems to increase over time. The opposite is true for G-SIBs since the error seems to decrease over time.

5.2 Binding constraints error due to incorrect data

As already said, the Basel Committee makes use of data that contain two sources of inaccuracy: uniform capital requirements and assets to exposure ratio equal to one. It is now natural to consider the impact of each type of inaccuracy separately on the aggregate error.

5.2.1 Errors due to the assets to exposure ratio assumption ($A = E$) only

As before, for each period of time, we compute the aggregate error and then compute the bank-specific error. For the case of December, 2018, the database is fully reported, which means that our results are easy to check.

Table 9: Overall mis-classification due to $A = E$ only.

Errors | Periods	December 2018	June 2019	December 2019	June 2020
Perc. (abs.)	15% (7)	4.2% (2)	6.4% (3)	8.5% (4)

When one compares Table 9 with Table 7, it is clear that considering the correct capital requirements of each bank when the assets to exposure is inaccurate considerably reduces the aggregate error. This aggregate error is at least divided by a factor of two and if one excludes December 2018, it is divided by a factor of at least four. The Basel committee would thus markedly reduce the error in the Basel III monitoring reports by using the minimum capital requirement applicable to each bank. It is interesting to note that the fact that the capital requirement is bank-dependent is a decision of the Basel Committee itself and constitutes an important ingredient of Basel III. It is fairly surprising that the Basel Committee does not follow its recommendation to determine the binding constraint.

Table 10: Mis-classification (by type of institution) due $A = E$ only

Errors | Periods	December 2018	June 2019	December 2019	June 2020
G-SIBs	30% (3)	0% (0)	10% (1)	20% (2)
G-SIIs	13% (3)	8.7% (2)	8.7% (2)	15.4%(2)
O-SIIs	7.1% (1)	0% (0)	0% (0)	0% (0)

Table 10 reveals that when one considers the correct capital requirements, for O-SIIs, the global error is equal to zero if one discards June 2020. For G-SIIs, the global error is considerably reduced since it is divided approximately by a factor of three or four. Surprisingly at first glance, in Table 10, two G-SIBs are incorrectly classified when data contains a single source of inaccuracy while there is no classification error for G-SIBs in Table 8 when they contain two sources of inaccuracy. To see how this can happen, consider the case of Lloyds Banking Group. From Table 5 (column uniform), the binding constraint is the leverage ratio (LR) while it is the risk-based capital ratio (RBC) when one considers bank-dependent capital requirements. From Table 13, it turns out that the correct binding constraint is the leverage ratio (LR) and not the risk-based capital ratio (RBC). From Table 5, we know that $\gamma = 30.81$. Since $\gamma_c^u = 35.29$, it thus follows from proposition 1 that the binding constraint is the leverage ratio. However, the correct capital requirement for that bank turns out to be much lower than the uniform requirement since it is equal to $\gamma_c = 29$. As a result, when one considers the correct capital requirements, from proposition 1, the binding constraint is the risk-based capital ratio (RBC) and an error is made. Part of the problem comes from the fact that the correct requirements (29) is much lower than the (wrong) uniform requirements (35.29). This analysis clearly reveals that one cannot expect the classification error to be an increasing function of the number of errors in the data (uniform requirements, assets to exposure ratio equal to one). As we shall see, this classification error can even decrease with the number of sources of inaccuracy.

5.2.2 Errors due to the uniform requirements assumption ($\gamma_c = 35.29\%$) only

We now consider the impact of the other source of inaccuracy only on the aggregate error, that is, when the capital requirements is assumed to be uniform across banks. It is important to point out that since we use the correct assets to exposure ratio (i.e., it is not equal to one), we know that the critical average risk weight approach is incorrect. As a result, if one wants to isolate the impact of the capital requirements on the aggregate error, one must use proposition 2 and not proposition 1.

Table 11: Overall mis-classification due to uniform requirements only.

Errors | Periods	December 2018	June 2019	December 2019	June 2020
Perc. (abs.)	25.5% (12)	40.5% (19)	38.3% (18)	38.3% (18)

When one compares Tables 9 and 11, it is clear that the assumption of a uniform capital requirement contributes to a global error which is much higher than the assumption of an assets to exposures ratio equal to one. As already said, for June, 2020, considering the correct assets to exposure even contributes to increasing the aggregate error. This error is

Table 12: Mis-classification (by type of institution) due to uniform requirements only

Errors | Periods	December 2018	June 2019	December 2019	June 2020
G-SIBs	10% (1)	40% (4)	40% (4)	10% (1)
G-SIIs	34.8% (8)	47.8% (11)	47.8% (11)	52.2% (12)
O-SIIs	21.4%(3)	17.4%(4)	21.4%(3)	35.7% (5)

equal to 38.3% (see Table 11) when only the capital requirements is incorrect while it is equal to 34% when both the capital requirements and the assets to exposure ratios are incorrect (see Table 7).

5.3 Binding constraints errors due to incorrect framework

We eventually turn to the aggregate error analysis when the unique source of inaccuracy is the framework used, that is, when one uses proposition 3 instead of proposition 2 with the correct data (correct capital requirements, correct assets to exposure ratio). It turns out that when one takes the critical average risk weight model with the correct data, the error due to the model is invariably equal to zero and this explains why we do not report any Table. As we shall now see, the error due to the model for realistic values of the parameters is very small, and this explains why the error due to the framework is nonexistent.

Recall that $x = \frac{A}{E}$ and let us now consider the error defined as $\zeta(x) := \widetilde{\gamma}_c - \overline{\gamma}_c \geq 0$ when $x \leq 1$. Using the fact that $\lambda = x\frac{K_1}{A}$, it is not difficult to show that

$$\zeta(x) = \gamma_c \times \left[\frac{(A - xK_1)}{x(A - K_1) + \lambda_{min} A(1 - x)} - \frac{1}{x} \right] \tag{35}$$

As expected, $\zeta(1) = 0$ but $\zeta(x) > 0$ when $x < 1$ and note that in general, ζ is a non-linear function that needs not be a decreasing function of x. For realistic values of $\frac{K}{A}$, the error turns out to be small even when x is fairly low or fairly high. To illustrate this, let us consider a numerical example with a fictitious bank. Assume for simplicity that $\lambda_{min} = 3\%$ and $\theta_{min} = 8.5\%$ so that $\gamma_c \approx 35.29\%$. From Tables 1 and 2, we know that Tier 1 capital in December 2018 is approximately equal to 5% of the total assets, that is, $\frac{K_1}{A} \approx 5\%$.[14] In this period, x varies from 0.76 (ING) to 1.12 (N.V. Bank Nederlandse Gemeenten) and the average value of x is equal to 0.95 for G-SIBs, 0.94 for G-SIIs and 0.96 for O-SIIs.

Consider a (fictitious) bank such that $A = 100$ and $K = 5$. When $x = 0.95$, using equation (35), $\zeta(0.95) = 0.3529 \times (1.536 - 1.526) \approx 0.00353$, the difference is negligible.

[14] The Tier 1 capital stands at 5.20% of the total assets for G-SIBs, 5.68% for G-SIIs and 7.24% for O-SIIs.

It is actually still negligible when x varies from 0.8 to 1.4. From equation (35), it is not difficult to see that this error increases with K_1. If one assumes that $A = 100$ and $K_1 = 40$ – a completely unrealistic highly-capitalized bank – for $x = 0.9$, we find an error equal to approximately 0.067, which is not anymore negligible. The critical average risk weight approach as given in proposition 3, although not correct from a theoretical point of view when A is not equal to E, gives an excellent approximation of the correct critical risk weight, at least when $\frac{K_1}{A}$ is low (around 5%). In that sense, the critical average risk weight framework, as developed in ESRB (2015, Annex 1), is robust when correct data are used.

6 Stress-testing a specific asset only

Up to now, we used our stress test framework to determine the binding constraint when *each asset* of the balance sheet was subject to the *same shock*. However, our framework can also be used to analyze the impact of a shock on a single asset j, that is, assuming that all the other assets are not shocked. Since supervisors can access granular information, they may consider an asset j with a regulatory weight α_j and determine the binding constraint when only this asset j is hit by a shock. We shall now illustrate how our approach can be used to determine the binding constraint when a class of assets subject to credit risk only are subject to the same shock.

Let $J_{credit} \subseteq J$ the subset of assets subject to credit risk and let $\alpha_j > 0$ be the relevant regulatory credit risk weight of asset $j \in J_{credit}$. This credit risk weight α_j may be computed by the bank using internal models (Internal Rating Based approach) and/or may simply be given by the standardized approach (SA). The credit risk-weighted assets of a bank are equal to

$$\text{RWA}_{credit} = \sum_{j \in J_{credit}} \alpha_j A_j \tag{36}$$

In practice, banks both make use of the internal rating based approach (IRBA) and of the standardized approach (SA) to compute those credit risk-weighted assets. In section 4.2 of its 2020 monitoring report, the Basel Committee (see BCBS, 2020) offers some descriptive statistics by asset class of the credit average risk weight, both under the standardized and the IRB approach. Overall, for internationally-active banks (Group 1), the average credit risk weight under the IRBA is equal to 34.4% (see p. 65) and thus is slightly lower than $\gamma_c \approx 35.29\%$. For the other banks (Group 2), this average credit risk weight is equal to 29.9%.

Let A be the total value of the assets defined $A = m + A_{credit} + A_{other}$, where A_{other} is

Table 13: Stress test framework for Large Banks. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based capital (resp. leverage) ratio.

	Uniform		Bank-Dependent	
Bank Name	$\gamma \times \Gamma_1$	Binding Constraint	$\gamma \times \Gamma_1$	Binding Constraint
Global Systemically Important Banks – G-SIBs				
HSBC Holdings Plc	33.04	LR	33.06	RBC
BNP Paribas	34.68	LR	34.69	LR
Groupe Crédit Agricole	33.46	LR	33.47	LR
Banco Santander S.A.	39.75	RBC	39.76	RBC
Barclays Plc	27.24	LR	27.24	LR
Deutsche Bank AG	27.55	LR	27.53	LR
Société Générale S.A.	31.12	LR	31.12	LR
Groupe BPCE	33.15	LR	33.15	LR
ING Groep N.V.	26.86	LR	26.91	LR
UniCredit S.p.A.	36.98	RBC	37.01	RBC
Global Systemically Important Institutions – G-SIIs				
Lloyds Banking Group Plc	28.94	LR	28.95	LR
Group Crédit Mutuel	36.77	RBC	36.77	RBC
The Royal Bank of Scotland Group Plc	29.34	LR	29.32	LR
Banco Bilbao Vizcaya Argentaria S.A.	49.24	RBC	49.24	RBC
Intesa Sanpaolo S.p.A.	41.28	RBC	41.28	RBC
Coöperatieve Rabobank U.A.	32.95	LR	32.95	RBC
Nordea Bank Group	29.50	LR	29.50	RBC
Commerzbank AG	37.07	RBC	37.07	RBC
ABN AMRO Group N.V.	21.82	LR	21.82	LR
DZ Bank AG	29.90	LR	29.90	RBC
Danske Bank	22.80	LR	22.80	RBC
CaixaBank S.A.	42.34	RBC	42.34	RBC
Svenska Handelsbanken Group	23.49	LR	23.49	RBC
Skandinaviska Enskilda Banken Group	25.71	LR	25.71	RBC
KBC Group NV	35.52	RBC	35.52	RBC
La Banque Postale	26.56	LR	26.56	LR
Landesbank Baden-Württemberg	30.83	LR	30.83	RBC
DNB Bank Group	41.02	RBC	41.12	RBC
Erste Group Bank AG	45.70	RBC	45.70	RBC
Bayerische Landesbank	27.17	LR	27.17	LR
Banco de Sabadell S.A.	36.22	RBC	36.22	RBC
Swedbank Group	28.40	LR	28.40	RBC
Nykredit Realkredit	22.41	LR	22.41	LR
Descriptive Statistics				
Mean	32.39		32.39	
G-SIBs	32.38		32.39	
G-SIIs	32.39		32.39	
Standard Deviation	6.82		6.82	
G-SIBs	4.28		4.28	
G-SIIs	7.75		7.76	

Table 14: Stress test framework for Smaller Banks. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based capital (resp. leverage) ratio.

	Uniform		Bank-Dependent	
Bank Name	$\gamma \times \Gamma_1$	Binding Constraint	$\gamma \times \Gamma_1$	Binding Constraint
Other Systemically Important Institutions – O-SIIs				
Landesbank Hessen-Thüringen Girozentrale	31.03	LR	31.03	RBC
Banco BPM S.p.A.	37.24	RBC	37.24	RBC
Norddeutsche Landesbank Girozentrale	26.88	LR	26.88	LR
Raiffeisen Bank International AG	44.27	RBC	44.27	RBC
Belfius Banque SA	35.09	LR	35.09	RBC
Unione di Banche Italiane S.p.A.	46.54	RBC	46.54	RBC
OP Financial Group	41.37	RBC	41.37	RBC
N.V. Bank Nederlandse Gemeenten	9.91	LR	9.91	LR
Bank of Ireland Group plc	43.63	RBC	43.63	RBC
Allied Irish Banks Group plc	54.69	RBC	54.69	RBC
Jyske Bank	29.46	LR	29.46	RBC
Powszechna Kasa Oszczednosci Bank Polski SA	59.29	RBC	59.29	RBC
OTP Bank Nyrt.	61.38	RBC	61.38	RBC
Bank Polska Kasa Opieki SA	60.94	RBC	60.94	RBC
Descriptive Statistics				
Mean	41.55		41.55	
Standard Deviation	14.75		14.75	

the value of the assets not subject to credit risk and let $\text{RWA} = \text{RWA}_{credit} + \text{RWA}_{other}$ be the total risk-weighted assets. Since we are only interested in assets subject to credit risk, RWA_{other} represents all the non-credit risk-weighted assets. In the annual reports of banks, RWA_{credit} is explicitly disclosed while A_{credit} is not explicitly disclosed as such. We must consequently infer A_{credit} from the information disclosed to investors. We reported the total assets (A) and the amount of cash (m) of our sample of European banks in Tables 1 and 2. The most important component of A_{credit} (not disclosed as such in annual reports) are the *financial assets at amortised cost*, a new classification of assets entered into force in 2018 with the adoption of the International Financial Reporting Standards (IFRS). Under IFRS, loans to customers, corporations or credit institutions are classified in the financial assets at amortised cost and are subject to credit risk. Those quantities are reported in Tables 15 and 16. In December 2018, the G-SIBs held on average 916.21 billion EUR of assets subject to credit risk. In comparison, those assets stood at an average of 303.10 billion EUR among the G-SIIs and 96.57 billion EUR for the O-SIIs. We also report in Tables 15 and 16 the value of the credit risk weighted assets (RWA_{credit}), where we include the risk weighted assets for settlement risk and for securitisation on exposures in the banking book. Credit risk weighted assets decrease similarly with the size of the bank: on average, they stand at 366.33 billion EUR for the G-SIBs, 111.88 billion EUR for G-SIIs and 38.94 billion EUR for the O-SIIs. Based on those quantities, one can thus imply α_{credit}, the average risk weight of A_{credit} as follows.

$$\alpha_{credit} := \alpha = \frac{\text{RWA}_{credit}}{A_{credit}} \tag{37}$$

Note importantly that this is once again an average weight since we do not have a direct access to the right hand side of equation (36). The average risk weight of the assets subject to credit risk is on average 39.72% among the G-SIBs and it is on average larger for smaller institutions: it increases to 44.69% for the G-SIIs and to 47.66% for the O-SIIs.

Let $\Delta_{credit} = \Delta \in [0, 1]$ be a shock in percentage on A_{credit} only so that the loss is equal to $A_{credit} \times \Delta$. It is easy to show that the capital ratios after the shock are equal to $\theta(\Delta) = \frac{K_1 - (A_{credit} \times \Delta)}{\gamma \times (A-m) - (A_{credit} \times \Delta \times \alpha)}$ and to $\lambda(\Delta) = \frac{K_1 - (A_{credit} \times \Delta)}{E - A_{credit} \Delta}$ so that the two critical shocks are equal to

$$\Delta^*_{credit,R} = \frac{K_1 - \theta_{min} \gamma (A - m)}{A_{credit}(1 - \alpha \theta_{min})} \tag{38}$$

$$\Delta^*_{credit,L} = \frac{K_1 - \lambda_{min} E}{A_{credit}(1 - \lambda_{min})} \tag{39}$$

In the following proposition, we realistically assume that $1 - \lambda - \alpha\theta_{min} > 0$. To obtain for instance equation (40), it suffices as before to solve $\Delta^*_{credit,R} < \Delta^*_{credit,L}$.

Proposition 5 *Let $\alpha > 0$ be the average credit risk regulatory weight and $\gamma > 0$ be the usual average risk weight.*

- *The Tier 1 risk-based capital ratio is the binding constraint if*

$$\gamma \times \underbrace{\left[\left(\frac{A-m}{E}\right)\left(\frac{1-\lambda_{min}}{1-\lambda-\alpha\theta_{min}}\right)\right] - \left(\frac{\alpha\lambda}{1-\lambda-\alpha\theta_{min}}\right)}_{\Gamma_3(.):=\Gamma_3} > \gamma_c \tag{40}$$

- *The leverage ratio is the binding constraint if*

$$\gamma \times \left[\left(\frac{A-m}{E}\right)\left(\frac{1-\lambda_{min}}{1-\lambda-\alpha\theta_{min}}\right)\right] - \left(\frac{\alpha\lambda}{1-\lambda-\alpha\theta_{min}}\right) < \gamma_c \tag{41}$$

When one considers a shock on the assets subject to credit risk, as the above proposition shows, two implied risk weights are required, γ, the average risk weight and α, the average credit risk weight. We report the results of proposition 5 in Tables 17 and 18. For comparability purposes with the previous cases examined above, we allow the capital requirements but also the assets to exposure ratio to vary across institutions. Using proposition 5, we examine the binding constraint from December 2018 to June 2020 in Figure 4 and we report the corresponding numerical values in Table 19. While proposition 5 is different from proposition 2 (and proposition 6 in the Apppendix), we find identical results in terms of binding constraints. Our conclusions are therefore unchanged when we stress test the assets that are exclusively subject to credit risk instead of all the assets.

Table 15: Assets Subject to Credit Risk for Large Banks. The reported period is December 2018. Non-credit assets (A_{other}) include (1) financial assets held for trading, (2) non-trading financial assets mandatorily at fair value through profit or loss, (3) financial assets designated at fair value through profit or loss and (4) derivatives (hedge accounting). Credit assets (A_{credit}) are the total assets minus cash and non-credit assets. Credit risk-weighted assets (RWA_{credit}) include risk weighted assets for settlement risk and for securitisation on exposures in the banking book. A_{other}, A_{credit} and RWA_{credit} are in billion EUR. α_{credit} is multiplied by 100.

Bank Name	Non-Credit Assets A_{other}	Credit Assets A_{credit}	Credit Risk-Weighted Assets RWA_{credit}	Average Risk Weight α_{credit}
Global Systemically Important Banks – G-SIBs				
HSBC Holdings Plc	399.79	1613.81	605.18	37.50
BNP Paribas	550.76	1082.09	522.55	48.29
Groupe Crédit Agricole	254.67	1179.74	456.45	38.69
Banco Santander S.A.	167.15	1165.12	495.28	42.51
Barclays Plc	535.28	530.52	218.83	41.25
Deutsche Bank AG	574.12	577.15	187.97	32.57
Société Générale S.A.	387.22	691.07	275.89	39.92
Groupe BPCE	208.67	876.62	332.96	37.98
ING Groep N.V.	120.49	714.35	259.02	36.26
UniCredit S.p.A.	91.38	731.64	309.21	42.26
Global Systemically Important Institutions – G-SIIs				
Lloyds Banking Group Plc	71.26	616.03	191.83	31.14
Group Crédit Mutuel	25.39	607.28	241.56	39.78
The Royal Bank of Scotland Group Plc	220.27	447.73	154.55	34.52
Banco Bilbao Vizcaya Argentaria S.A.	96.57	502.26	289.74	57.69
Intesa Sanpaolo S.p.A.	45.66	536.66	229.49	42.76
Coöperatieve Rabobank U.A.	27.83	488.18	165.87	33.98
Nordea Bank Group	156.67	310.58	114.31	36.81
Commerzbank AG	77.10	331.23	137.42	41.49
ABN AMRO Group N.V.	8.48	336.88	73.68	21.87
DZ Bank AG	51.37	313.34	106.43	33.97
Danske Bank	211.87	192.78	78.86	40.91
CaixaBank S.A.	20.03	296.50	128.19	43.23
Svenska Handelsbanken Group	22.93	223.58	44.17	19.75
Skandinaviska Enskilda Banken Group	36.99	161.52	50.08	31.00
KBC Group NV	6.77	226.69	75.58	33.34
La Banque Postale	11.40	228.76	57.56	25.16
Landesbank Baden-Württemberg	31.58	180.86	65.56	36.25
DNB Bank Group	44.21	180.45	87.33	48.39
Erste Group Bank AG	8.58	210.13	91.05	43.33
Bayerische Landesbank	13.77	199.63	55.38	27.74
Banco de Sabadell S.A.	2.49	196.28	68.35	34.82
Swedbank Group	15.09	169.34	29.43	17.38
Nykredit Realkredit	176.02	14.61	36.90	252.45
Descriptive Statistics				
Mean	141.57	488.89	188.99	43.18
G-SIBs	328.95	916.21	366.33	39.72
G-SIIs	60.10	303.10	111.88	44.69
Standard Deviation	168.49	362.72	152.62	38.47
G-SIBs	184.50	339.34	142.92	4.24
G-SIIs	66.86	158.65	70.80	46.23

Table 16: Assets Subject to Credit Risk for Smaller Banks. The reported period is December 2018. Non-credit assets (A_{other}) include (1) financial assets held for trading, (2) non-trading financial assets mandatorily at fair value through profit or loss, (3) financial assets designated at fair value through profit or loss and (4) derivatives (hedge accounting). Credit assets (A_{credit}) are the total assets minus cash and non-credit assets. Credit risk-weighted assets (RWA_{credit}) include risk-weighted assets for settlement risk and for securitisation on exposures in the banking book. A_{other}, A_{credit}, RWA_{credit} are in billion EUR. α_{credit} is multiplied by 100.

Bank Name	Non-Credit Assets A_{other}	Credit Assets A_{credit}	Credit Risk-Weighted Assets RWA_{credit}	Average Risk Weight α_{credit}
Other Systemically Important Institutions – O-SIIs				
Landesbank Hessen-Thüringen Girozentrale	21.92	133.00	45.38	34.12
Banco BPM S.p.A.	5.84	152.24	55.36	36.36
Norddeutsche Landesbank Girozentrale	12.85	141.53	35.01	24.73
Raiffeisen Bank International AG	8.09	109.20	60.09	55.03
Belfius Banque SA	15.72	122.63	42.01	34.26
Unione di Banche Italiane S.p.A.	1.28	112.37	55.08	49.02
OP Financial Group	4.15	104.96	41.08	39.14
N.V. Bank Nederlandse Gemeenten	10.00	125.93	8.72	6.92
Bank of Ireland Group plc	2.42	99.00	42.06	42.48
Allied Irish Banks Group plc	1.31	82.86	45.30	54.67
Jyske Bank	57.59	20.78	20.23	97.37
Powszechna Kasa Oszczednosci Bank Polski SA	1.21	66.91	42.94	64.17
OTP Bank Nyrt.	0.72	40.05	24.92	62.22
Bank Polska Kasa Opieki SA	0.67	40.50	27.02	66.72
Descriptive Statistics				
Mean	10.27	96.57	38.94	47.66
Standard Deviation	15.09	40.78	14.46	21.84

Table 17: Stress test Framework for Assets Subject to Credit Risk for Large Banks with Bank-Dependent Capital Requirements. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based (resp. leverage) ratio.

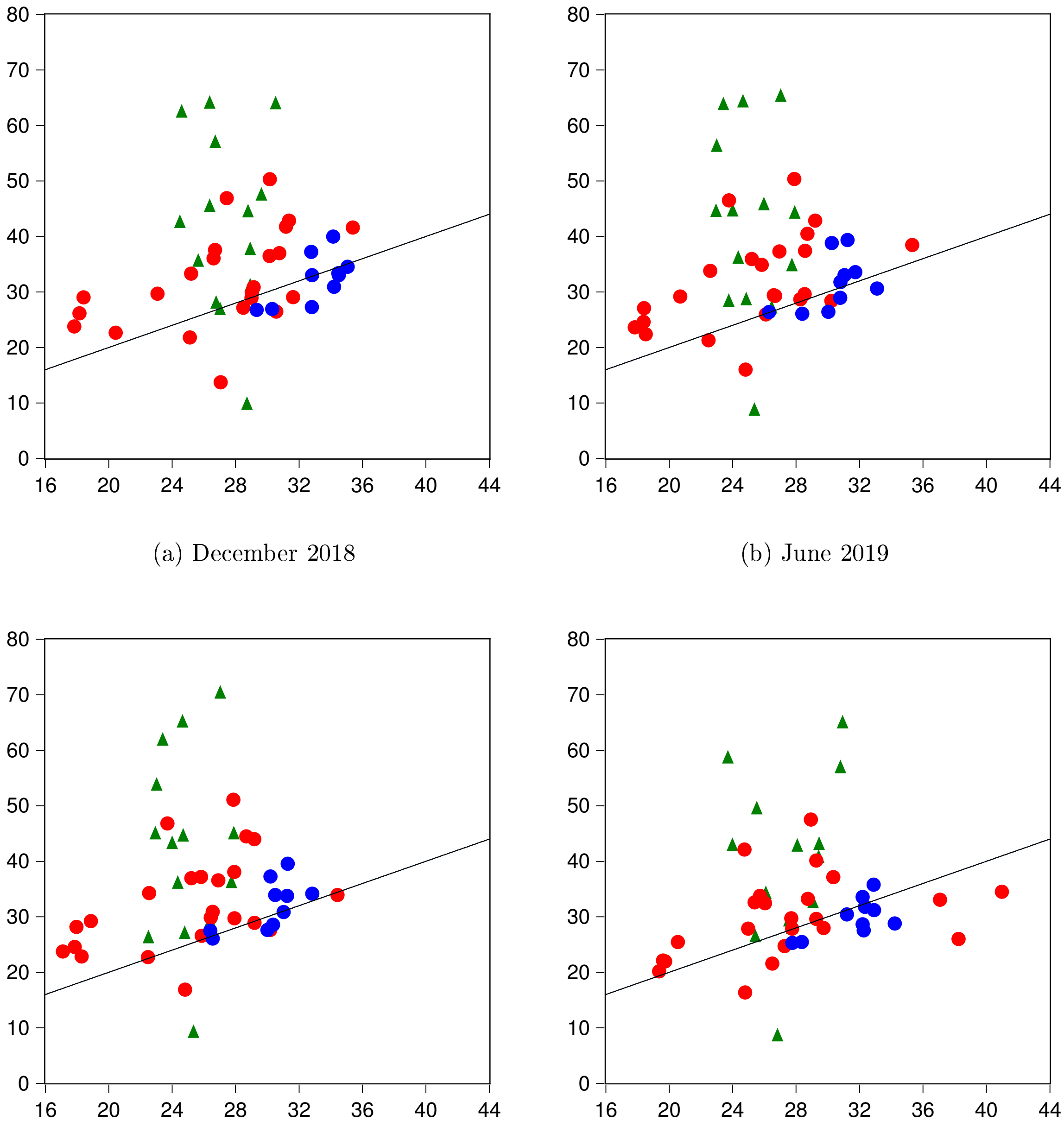
Bank Name	$\gamma \times \Gamma_3$	Binding Constraint
Global Systemically Important Banks – G-SIBs		
HSBC Holdings Plc	33.02	RBC
BNP Paribas	34.55	LR
Groupe Crédit Agricole	33.37	LR
Banco Santander S.A.	40.00	RBC
Barclays Plc	26.95	LR
Deutsche Bank AG	27.28	LR
Société Générale S.A.	30.93	LR
Groupe BPCE	33.01	LR
ING Groep N.V.	26.78	LR
UniCredit S.p.A.	37.24	RBC
Global Systemically Important Institutions – G-SIIs		
Lloyds Banking Group Plc	28.94	LR
Group Crédit Mutuel	36.99	RBC
The Royal Bank of Scotland Group Plc	29.06	LR
Banco Bilbao Vizcaya Argentaria S.A.	50.30	RBC
Intesa Sanpaolo S.p.A.	41.77	RBC
Coöperatieve Rabobank U.A.	33.30	RBC
Nordea Bank Group	29.69	RBC
Commerzbank AG	37.56	RBC
ABN AMRO Group N.V.	21.80	LR
DZ Bank AG	29.89	RBC
Danske Bank	22.66	RBC
CaixaBank S.A.	42.86	RBC
Svenska Handelsbanken Group	23.77	RBC
Skandinaviska Enskilda Banken Group	26.15	RBC
KBC Group NV	36.04	RBC
La Banque Postale	26.48	LR
Landesbank Baden-Württemberg	30.84	RBC
DNB Bank Group	41.61	RBC
Erste Group Bank AG	46.89	RBC
Bayerische Landesbank	27.16	LR
Banco de Sabadell S.A.	36.48	RBC
Swedbank Group	29.02	RBC
Nykredit Realkredit	13.74	LR
Descriptive Statistics		
Mean	32.31	
G-SIBs	32.31	
G-SIIs	32.30	
Standard Deviation	7.60	
G-SIBs	4.44	
G-SIIs	8.72	

Table 18: Stress test Framework for Assets Subject to Credit Risk for Smaller Banks with Bank-Dependent Capital Requirements. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based capital (resp. leverage) ratio.

Bank Name	$\gamma \times \Gamma_3$	Binding Constraint
Other Systemically Important Institutions – O-SIIs		
Landesbank Hessen-Thüringen Girozentrale	31.10	RBC
Banco BPM S.p.A.	37.65	RBC
Norddeutsche Landesbank Girozentrale	26.84	LR
Raiffeisen Bank International AG	45.43	RBC
Belfius Banque SA	35.55	RBC
Unione di Banche Italiane S.p.A.	47.49	RBC
OP Financial Group	42.55	RBC
N.V. Bank Nederlandse Gemeenten	9.78	LR
Bank of Ireland Group plc	44.46	RBC
Allied Irish Banks Group plc	56.96	RBC
Jyske Bank	27.98	RBC
Powszechna Kasa Oszczednosci Bank Polski SA	62.43	RBC
OTP Bank Nyrt.	63.92	RBC
Bank Polska Kasa Opieki SA	64.01	RBC
Descriptive Statistics		
Mean	42.58	
Standard Deviation	15.92	

This methodology used for assets subject to credit risk could obviously be replicated for each class of assets. For instance, one could try to follow a similar analysis for the assets subject to market risk or to counterparty risk. Let J_{market} be the subset of J such that A_j is subject to market risk. As recalled in BCBS (2019a) (see Section RBC20, p. 142), the risk-weighted assets (RWA) for market risk (under the standardised approach or under internal models) are determined by multiplying the calculated capital requirements by 12.5 (i.e., $\frac{1}{8\%}$). If K_j denotes the Tier 1 capital required for asset $j \in J_{market}$, it thus follows that $\text{RWA}_j = 12.5 \times K_j$. Writing $\text{RWA}_j = \alpha_j A_j$, the market risk weight is equal to $\alpha_j = \frac{12.5 \times K_j}{A_j}$.

Such an analysis still remains at an aggregate level. Since regulators have possibly access to more granular information, an interesting application of our single asset stress-testing methodology could be to consider derivatives or even a particular class of derivatives, e.g., credit derivatives or interest rate derivatives. Such products are more complex than loans since they both appear on the asset side and on the liability side of the balance sheet, which raises new issues. If one succeeds to perform such an analysis, one ends up with the



(a) December 2018

(b) June 2019

(c) December 2019

(d) June 2020

Figure 4: Stress test framework for Assets Subject to Credit Risk with Bank-Dependent Capital Requirements – Evolution over Time. This Figure shows the evolution of the results from proposition 5 over time, when capital requirements vary across banks. The period is half-yearly and is from December 2018 to June 2020. Γ_5 is in the vertical axis and γ_c is in the horizontal axis. Blue (red) dots denote the G-SIBs (G-SIIs). Green triangles depict the O-SIIs.

Table 19: Stress test Framework for Assets Subject to Credit Risk with Bank-Dependent Capital Requirements – December 2018 to June 2020. The period is half yearly. G-SIBs is for the global systemically important banks. G-SIIs is for the global systemically important institutions. O-SIIs is for the other systemically important institutions. RBC (resp. LR) denotes the number of banks that are constrained by the risk-based capital (resp. leverage) ratio.

		December 2018		June 2019		December 2019		June 2020	
	Total	RBC	LR	RBC	LR	RBC	LR	RBC	LR
G-SIBs	10	3	7	6	4	6	4	2	8
G-SIIs	23	17	6	19	4	19	4	16	7
O-SIIs	14	12	2	13	1	13	1	13	1
Total	47	32	15	38	9	38	9	31	16

binding constraint for say a specific class of derivatives, which may be something valuable to regulators.

7 Concluding remarks and policy implications

We presented in this paper a fairly general stress-test based methodology to determine the Tier 1 capital ratio which is the binding constraint. From a theoretical point of view, we have shown that the critical average risk weight model used by regulatory authorities such as the BoE, the ESRB or the Basel Committee in its regular Basel III monitoring reports is incorrect when the assets to exposure ratio is different from one. From an empirical point of view, we have shown that by considering two different types of inaccuracy in the data used (uniform capital requirements and assets to exposure ratio equal to one), the Basel Committee makes an important error regarding the capital ratio which is the binding constraint. For each period, at least one third of banks are incorrectly classified. The two types of inaccuracy in data used by the Basel Committee are however not symmetric. We have shown that considering a uniform capital requirement leads to an aggregate error which is much higher than considering an assets to exposure ratio equal to one.

Overall, our results suggest that the Basel Committee could sharply reduce the error in its Basel III monitoring reports by simply considering the correct bank-dependent capital requirements, using still the critical average risk weight model. Whenever the assets to exposure ratio is equal to one, the model used to determine the binding constraint is correct. However, when this ratio is different from one, the critical average risk weight model is not anymore correct. Interestingly, for realistic values of the parameters, the error attributable to the model in terms of critical weights is negligible and does not alter the binding constraint. As a result, when the critical average risk weight model is used but there is no inaccuracy in the data, the error completely disappears. In that sense, the critical average risk weight model is robust.

Our stress-test approach can also be used to determine the binding constraint when only one asset is shocked. This means for instance that the Basel Committee could make use of our framework and use granular information to determine the binding constraint when one (or more than one) derivative asset is shocked. We provided an application to the banking book, interpreted as the subset of assets in the balance sheet subject to credit risk only. To perform the analysis, we derived from annual reports the implied regulatory credit risk weight and we have shown that this leaves our empirical results unchanged in terms of binding constraint. In some sense, these results are not so surprising since credit is by far the most important risk for banks.

Appendix

Proofs

Proof of lemma 1. Recall that $\Delta_0 < 1$ is the smallest shock size for which the numerator of the two capital ratios is equal to zero, that is, $\lambda(\Delta_0) = \theta(\Delta_0) = 0$. Since the functions $\lambda(\Delta)$ and $\theta(\Delta)$ are decreasing function of Δ, for all $\Delta \in [0, \Delta_0)$, it is true that $\lambda(\Delta) > 0$ and $\theta(\Delta) > 0$. Let Δ_R^* be the smallest critical threshold such that $\theta(\Delta_R^*) = \theta_{min}$. Since $\theta > \theta_{min}$ and since equation (22) is a continuous and strictly decreasing function of $\Delta \in (0, \Delta_0)$, there exists a unique Δ_R^* such that $\theta(\Delta_R^*) = \theta_{min}$. It suffices now to solve $\theta(\Delta_R^*) = \theta_{min}$ in Δ_R^* to obtain equation (24). One easily finds Δ_L^* as given in equation ((25)) in exactly the same way □

Proof of proposition 2. Using equations (24) and (25), it is easy to show that $\Delta_R^* < \Delta_L^*$ is equivalent to $\gamma \times \left(\frac{\lambda_{min}(E-A)+(A-K_1)}{E-K_1}\right) > \gamma_c$. By dividing both the numerator and the denominator by E in $\left(\frac{\lambda_{min}(E-A)+(A-K_1)}{E-K_1}\right)$, we obtain that $\frac{\lambda_{min}(1-\frac{A}{E})+\frac{A}{E}-\lambda}{1-\lambda} = \frac{A}{E}\left(\frac{1-\lambda_{min}}{1-\lambda}\right) - \left(\frac{\lambda-\lambda_{min}}{1-\lambda}\right)$ and this proves inequality (28). Reverse the inequality to obtain inequality (29) □

Additional data

Table 20: Risk-Based Capital Requirements for Large Banks. The reported period is December 2018. CET1 stands for common equity Tier 1. AT1 is additional Tier 1. CCoB is the capital conservation buffer. SyRB is the systemic risk buffer. CCyB is the counter-cyclical buffer. All percentages are multiplied by 100.

	Pillar 1		Pillar 2		Buffers		
Bank Name	CET1	AT1	CET1	AT1	CCoB	SyRB	CCyB
Global Systemically Important Banks – G-SIBs							
HSBC Holdings Plc	4.50	1.50	1.60	0.50	1.875	1.50	0.56
BNP Paribas	4.50	1.50	1.25	–	1.875	1.50	0.07
Groupe Crédit Agricole	4.50	1.50	1.50	–	1.875	0.75	0.03
Banco Santander S.A.	4.50	1.50	1.50	–	1.875	0.75	0.12
Barclays Plc	4.50	1.50	2.40	0.80	1.875	1.10	0.50
Deutsche Bank AG	4.50	1.50	2.75	–	1.875	1.50	0.07
Société Générale S.A.	4.50	1.50	1.50	–	1.875	0.75	0.11
Groupe BPCE	4.50	1.50	1.50	–	1.875	0.75	0.02
ING Groep N.V.	4.50	1.50	1.75	–	1.875	2.25	0.06
UniCredit S.p.A.	4.50	1.50	2.00	–	1.875	0.75	0.06
Global Systemically Important Institutions – G-SIIs							
Lloyds Banking Group Plc	4.50	1.50	2.60	0.87	1.875	0.00	0.90
Group Crédit Mutuel	4.50	1.50	1.50	–	1.875	0.38	0.00
The Royal Bank of Scotland Group Plc	4.50	1.50	2.10	0.70	1.875	0.80	0.70
Banco Bilbao Vizcaya Argentaria S.A.	4.50	1.50	1.50	–	1.875	0.56	0.01
Intesa Sanpaolo S.p.A.	4.50	1.50	1.50	–	1.875	0.19	0.06
Coöperatieve Rabobank U.A.	4.50	1.50	1.75	–	1.875	2.25	0.03
Nordea Bank Group	4.50	1.50	3.60	–	2.500	0.00	0.90
Commerzbank AG	4.50	1.50	2.25	–	1.875	1.00	0.11
ABN AMRO Group N.V.	4.50	1.50	1.75	–	1.875	2.25	0.07
DZ Bank AG	4.50	1.50	1.75	–	1.875	0.66	0.05
Danske Bank	4.50	1.50	2.70	0.50	1.875	2.40	1.20
CaixaBank S.A.	4.50	1.50	1.50	–	1.875	0.19	0.00
Svenska Handelsbanken Group	4.50	1.50	3.60	0.23	2.500	3.00	1.50
Skandinaviska Enskilda Banken Group	4.50	1.50	3.60	0.23	2.500	3.00	1.20
KBC Group NV	4.50	1.50	1.75	–	1.875	1.50	0.15
La Banque Postale	4.50	1.50	1.75	–	1.875	0.19	0.00
Landesbank Baden-Württemberg	4.50	1.50	1.75	–	1.875	0.67	0.00
DNB Bank Group	4.50	1.50	1.80	–	2.500	5.00	1.65
Erste Group Bank AG	4.50	1.50	1.75	–	1.875	1.00	0.31
Bayerische Landesbank	4.50	1.50	2.00	–	1.875	0.66	0.00
Banco de Sabadell S.A.	4.50	1.50	1.75	–	1.875	0.19	0.14
Swedbank Group	4.50	1.50	3.10	0.10	2.500	3.00	1.60
Nykredit Realkredit	4.50	1.50	1.13	0.38	1.875	1.60	0.10
Descriptive Statistics							
Mean	4.50	1.50	2.01	0.48	1.970	1.28	0.37
G-SIBs	4.50	1.50	1.78	0.65	1.875	1.16	0.16
G-SIIs	4.50	1.50	2.11	0.43	2.011	1.33	0.46
Standard Deviation	0.00	0.00	0.68	0.27	0.228	1.11	0.52
G-SIBs	0.00	0.00	0.47	0.21	0.000	0.51	0.20
G-SIIs	0.00	0.00	0.73	0.28	0.264	1.29	0.59

Table 21: Risk-Based Capital Requirements for Smaller Banks. The reported period is December 2018. CET1 stands for common equity Tier 1. AT1 is additional Tier 1. CCoB is the capital conservation buffer. SyRB is the systemic risk buffer. CCyB is the counter-cyclical buffer. All percentages are multiplied by 100. * is the value reported in June 2019.

	Pillar 1		Pillar 2		Buffers		
Bank Name	CET1	AT1	CET1	AT1	CCoB	SyRB	CCyB
Other Systemically Important Institutions – O-SIIs							
Landesbank Hessen-Thüringen Girozentrale	4.50	1.50	1.75	–	1.875	0.66	0.09
Banco BPM S.p.A.	4.50	1.50	2.50	–	1.875	0.00	0.00
Norddeutsche Landesbank Girozentrale	4.50	1.50	2.50	–	1.875	0.66	0.07
Raiffeisen Bank International AG	4.50	1.50	2.25	–	1.875	1.00	0.25
Belfius Banque SA	4.50	1.50	2.25	–	1.875	1.50	0.07
Unione di Banche Italiane S.p.A.	4.50	1.50	2.25	–	1.875	0.00	0.00
OP Financial Group	4.50	1.50	1.75	–	2.500	2.00	0.00
N.V. Bank Nederlandse Gemeenten	4.50	1.50	1.75	–	1.875	0.75	0.08
Bank of Ireland Group plc	4.50	1.50	2.25	–	1.875	0.00	0.30
Allied Irish Banks Group plc	4.50	1.50	3.15	–	1.875	0.00	0.20
Jyske Bank	4.50	1.50	1.60	0.53	1.875	1.20	0.00
Powszechna Kasa Oszczednosci Bank Polski SA	4.50	1.50	0.00	–	1.875	4.31	0.01
OTP Bank Nyrt.	4.50	1.50	0.70*	0.20	1.875	1.00	0.05
Bank Polska Kasa Opieki SA	4.50	1.50	0.01	–	1.875	3.48	0.01
Descriptive Statistics							
Mean	4.50	1.50	1.77	0.37	1.920	1.18	0.08
Standard Deviation	4.50	1.50	0.93	0.23	0.167	1.31	0.10

Robustness analysis: Taking cash into account

Up to now, we made the assumption that the total value of the assets A is equal to the total value of the risky assets, that is, $A = \sum_{j \in J} A_j$. However, partly due to the new Basel III liquidity ratios, banks now hold a non-negligible fraction of cash on their balance sheets. As reported in Tables 1 and 2, the amount of cash held by our sample of European banks ranged from 1.50 billion EUR (Norddeutsche Landesbank Girozentrale) to 198.63 billion EUR (Barclays). This consequently means that the value of the risky assets is lower than the total value of the assets. With positive cash, $A = \sum_{j \in J} A_j + m$ and the total value of the assets after a shock Δ (on the risky assets) is equal to $A(\Delta) = \sum_{j \in J} A_j(1-\Delta) + m = (A-m)(1-\Delta) + m$ where $(A-m)\Delta$ is the loss. It is easy to show that the capital ratios after the shocks are given by

$$\theta(\Delta) = \frac{K_1 - (A-m)\,\Delta}{\gamma(A-m)(1-\Delta)} \tag{42}$$

$$\lambda(\Delta) = \frac{K_1 - (A-m)\,\Delta}{E - (A-m)\Delta} \tag{43}$$

Compared to the no-cash case (see equations (22) and (23)), A is simply replaced with $A-m$ so that this is also true in lemma 1 and thus in proposition 2. As a result, with cash, the following result is true.

Proposition 6

- *The Tier 1 risk-based capital ratio is the binding constraint if*

$$\gamma \times \underbrace{\left[\left(\frac{A-m}{E}\right)\left(\frac{1-\lambda_{min}}{1-\lambda}\right) - \left(\frac{\lambda-\lambda_{min}}{1-\lambda}\right)\right]}_{\Gamma_2\left(\frac{A}{E},\lambda_{min},\lambda,m\right):=\Gamma_2} > \gamma_c \tag{44}$$

- *The leverage ratio is the binding constraint if*

$$\gamma \times \left[\left(\frac{A-m}{E}\right)\left(\frac{1-\lambda_{min}}{1-\lambda}\right) - \left(\frac{\lambda-\lambda_{min}}{1-\lambda}\right)\right] < \gamma_c \tag{45}$$

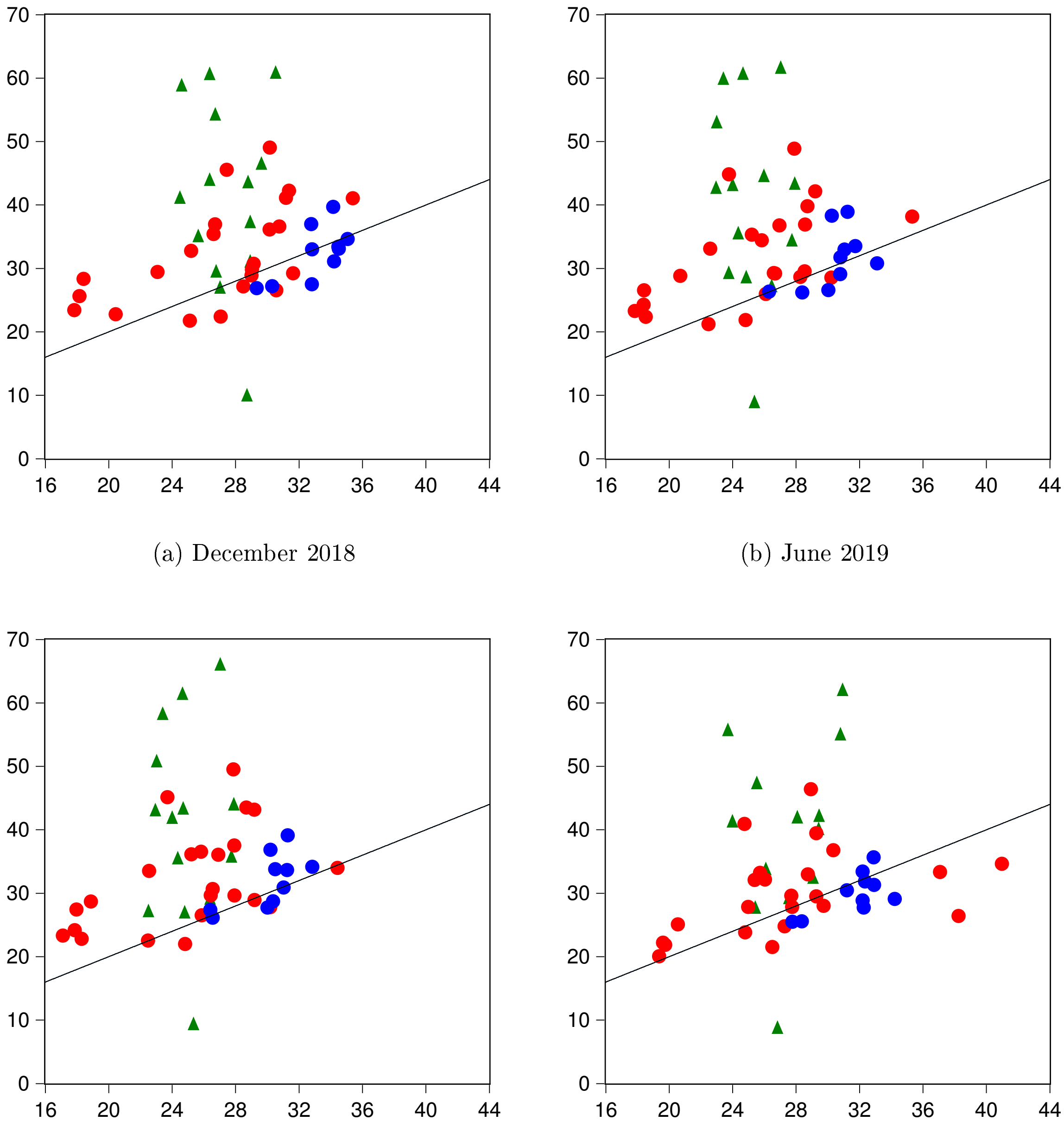
We examine the results of proposition 6 in Tables 22 and 23. We report $\gamma \times \Gamma_2$ for our sample of banks along with the corresponding binding constraints. The reported period is December 2018. First, we observe that accounting for the amount of cash held by financial institutions does not alter the conclusions that we obtained with the case in which $m = 0$. Second, compared to proposition 2, the left-hand side of proposition 6 yields values that are on average lower, but also less dispersed across banks.

Table 22: Stress test framework with Cash for Large Banks with Bank-Dependent Capital Requirements. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based capital (resp. leverage) ratio.

Bank Name	$\gamma \times \Gamma_2$	Binding Constraint
Global Systemically Important Banks – G-SIBs		
HSBC Holdings Plc	33.01	RBC
BNP Paribas	34.65	LR
Groupe Crédit Agricole	33.43	LR
Banco Santander S.A.	39.69	RBC
Barclays Plc	27.20	LR
Deutsche Bank AG	27.52	LR
Société Générale S.A.	31.10	LR
Groupe BPCE	33.10	LR
ING Groep N.V.	26.89	LR
UniCredit S.p.A.	36.98	RBC
Global Systemically Important Institutions – G-SIIs		
Lloyds Banking Group Plc	28.90	LR
Group Crédit Mutuel	36.61	RBC
The Royal Bank of Scotland Group Plc	29.23	LR
Banco Bilbao Vizcaya Argentaria S.A.	49.05	RBC
Intesa Sanpaolo S.p.A.	41.13	RBC
Coöperatieve Rabobank U.A.	32.78	RBC
Nordea Bank Group	29.43	RBC
Commerzbank AG	36.97	RBC
ABN AMRO Group N.V.	21.78	LR
DZ Bank AG	29.82	RBC
Danske Bank	22.79	RBC
CaixaBank S.A.	42.27	RBC
Svenska Handelsbanken Group	23.44	RBC
Skandinaviska Enskilda Banken Group	25.63	RBC
KBC Group NV	35.42	RBC
La Banque Postale	26.56	LR
Landesbank Baden-Württemberg	30.74	RBC
DNB Bank Group	41.08	RBC
Erste Group Bank AG	45.55	RBC
Bayerische Landesbank	27.16	LR
Banco de Sabadell S.A.	36.14	RBC
Swedbank Group	28.34	RBC
Nykredit Realkredit	22.40	LR
Descriptive Statistics		
Mean	32.33	
G-SIBs	32.36	
G-SIIs	32.31	
Standard Deviation	6.79	
G-SIBs	4.27	
G-SIIs	7.72	

Table 23: Stress test framework with Cash for Smaller Banks with Bank-Dependent Capital Requirements. The reported period is December 2018. All percentages are multiplied by 100. RBC (resp. LR) denotes a bank that is constrained by the risk-based capital (resp. leverage) ratio.

Bank Name	$\gamma \times \Gamma_2$	Binding Constraint
Other Systemically Important Institutions – O-SIIs		
Landesbank Hessen-Thüringen Girozentrale	31.00	RBC
Banco BPM S.p.A.	37.23	RBC
Norddeutsche Landesbank Girozentrale	26.88	LR
Raiffeisen Bank International AG	43.87	RBC
Belfius Banque SA	35.03	RBC
Unione di Banche Italiane S.p.A.	46.43	RBC
OP Financial Group	41.08	RBC
N.V. Bank Nederlandse Gemeenten	9.90	LR
Bank of Ireland Group plc	43.52	RBC
Allied Irish Banks Group plc	54.19	RBC
Jyske Bank	29.44	RBC
Powszechna Kasa Oszczednosci Bank Polski SA	58.78	RBC
OTP Bank Nyrt.	60.76	RBC
Bank Polska Kasa Opieki SA	60.56	RBC
Descriptive Statistics		
Mean	41.33	
Standard Deviation	14.56	



(a) December 2018

(b) June 2019

(c) December 2019

(d) June 2020

Figure 5: Stress test framework with Cash and with Bank-Dependent Capital Requirements – Evolution over Time. This Figure shows the evolution of the results from proposition 4 over time, when capital requirements vary across banks. The period is half-yearly and is from December 2018 to June 2020. $\gamma \times \Gamma_2$ is in the vertical axis and γ_c is in the horizontal axis. Blue (red) dots denote the G-SIBS (G-SIIs). Green triangles depict the O-SIIs.

Table 24: Stress test framework with Cash and with Bank-Dependent Capital Requirements – December 2018 to June 2020. The period is half yearly. G-SIBs is for the global systemically important banks. G-SIIs is for the global systemically important institutions. O-SIIs is for the other systemically important institutions. RBC (resp. LR) denotes the number of banks that are constrained by the risk-based capital (resp. leverage) ratio.

		December 2018		June 2019		December 2019		June 2020	
	Total	RBC	LR	RBC	LR	RBC	LR	RBC	LR
G-SIBs	10	3	7	6	4	6	4	2	8
G-SIIs	23	17	6	19	4	19	4	16	7
O-SIIs	14	12	2	13	1	13	1	13	1
Total	47	32	15	38	9	38	9	31	16

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