Credit Risk Models

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Abstract

We want to dispute in this article the idea that structural models are used just as intensity models, that is, as a "probabilistic black box" designed to be calibrated on market data such as the credit spreads of CDS. We begin this paper by offering a quick introduction to the CDS market, and then present a selected review of some existing models (intensity and structural) to compute the mark-to-market of a CDS. Calibration algorithms are also discussed. We then present a non representative sample of recent models from quantitative corporate that aim to provide a more realistic explanation of the default process.

Keywords: CDS, valuation models, structural models.

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1 Introduction

Most quantitative models used for the valuation of (exotic) credit derivatives are essentially used as "probabilistic black box" devoted to be calibrated on market data (i.e., vanilla quotes) rather than on corporate one. The main reason is that if vanilla markets are arbitrage-free, by calibrating the model using vanilla quotes, one can price exotics in an arbitrage-free way. Due to this practice, the problem is not to construct a realistic model, but a simple black-box in which the default probability (e.g., for a CDS) can be computed. Credit risk models are generally classified as *intensity* or *structural*.

- Intensity models, which are explicitly designed to be a (convenient) parametric black box for *relative valuation*, i.e., they do not allow to determine whether or not the spread is "fair".
- Structural models, which are intended to give a realistic picture of the actual default process of the firm. In principle, one can determine the fundamental (or fair) value of the spread.

In both models, default arises when a stochastic process hits a given threshold. In structural models, default is declared when the value of the assets V falls below a barrier H which is generally related to the face value of the debt. While V and H could in principle be estimated using accounting data, the triggering default mechanism is (implicitly) considered as too simple to be realistic. As a consequence, V and H are implied from market data so that structural models are used as parametric black box designed for relative valuation, just like intensity models. This is the idea we want to dispute in this paper.

The 2007 financial crisis has revealed that market liquidity can be sharply (and suddenly) reduced so that many credit derivatives can not be anymore priced since their price are only *relative*. Beside models designed for calibration, we believe important to develop and estimate properly quantitative models from corporate (i.e., accounting) data *solely* to get a "fundamental" price. This is not an easy task not only because corporate data are at best available on a quarterly basis and may depend on accounting policy, but also because the real default process may (sometimes) depend on non observables factors (e.g., bank power for short-term debt). In a recent paper, Das et al (2008) argue that corporate data is relevant in bankruptcy prediction even without the inclusion of market data, but is also value-relevant to users of credit derivatives.

This paper, which focuses on single name (while our discussion holds for multi name) is organized as follows. In the first part, we briefly review the CDS market, and then present some existing models used to compute the mark-to-market. We begin with purely non financial ones and finish with (a selected review of) the most financial ones. In the second part, we present the three level of fair valuation suggested by Crouhy, Jarrow and Turnbull (2008), and we offer a presentation of some recent models that are designed to give a realistic explanation of the default process. A clear and important distinction between *liquidity* and *solvency* is made. While this literature is still burgeoning, it is a first step toward a better understanding of the actual default process.

2 How to extract credit models' parameters from CDS data?

2.1 A short introduction to CDS

A Credit Default Swap is a contingent claim that transfers the loss due to a credit event (e.g., default) from one counterparty to another.

- The buyer of protection pays on a regular basis the premium as long as the credit event is not declared.
- In exchange, the seller of protection will make a payment contingent on the credit event[1].

The CDS contract specifies the underlying debt issuer (e.g., the society XYZ) the fixed (contractual) spread S_T^c which is generally expressed in basis point (T is the maturity), the nominal or a notional N, the maturity of the contract T, the payment schedule, and finally a contractual recovery rate. Let τ be the default time, which is indeed the only unknown quantity[2]. The CDS works as follows. As long as default is not declared, the buyer of protection pays say on a quarterly basis, the premium equal to $(1/4)S_T^c N$ to the seller of protection. When default is declared before T (if ever), the seller of protection pays the credit loss equal to $(1-R)N$ to the buyer of protection. While a CDS may be used for many purposes (speculation, regulatory, capital

[1] In practice, the seller of protection may also default so that there may exist some counterparty risk.

[2] This is not exactly true since the *observed* recovery rate may be different of the contractual recovery rate.

structure arbitrage...), its initial aim was to be an insurance-like contract which is a protection against the losses due to the default of some debt issuer.

The CDS market quotes the *market spread* of many underlying debt issuers for some standard maturities, where the market spread (at some point in time) is the spread at which an investor can buy or sell protection. Let S_{T_k} be the market spread of a CDS of maturity T_k at time $t = 0$, and let $\mathcal{T} = \{1Y, 2Y, 3Y, 5Y, 7Y, 10Y\}$ be the set of standard maturities. Since there is no payment to enter in a CDS position, its mark-to-market must be equal to zero at the market spread[3].

$$MtM(S_{T_k}) = 0 \qquad T_k \in \mathcal{T} \tag{1}$$

The mark-to-market of the CDS splits in two legs:

- the *premium leg*, which is the expected discounted value of the payment of the protection buyer.
- the *protection leg*, which is the expected discounted value of the payment of the protection seller.

Recently, a new form of CDS has been introduced. The contract is now expressed as an upfront payment U, plus a *fixed spread*, 100 bps for investment grade CDS, and 500 bps for high yield CDS. The recovery rate, depending on the credit quality, is either 20% or 40%. Since one can convert the upfront quotes to a running one[4], we shall work in what follows with a running spread.

Once we are given a *probabilistic model* $\mathbb{P}$ that allows to compute say the default probability $\mathbb{P}(\tau \le t)$, assuming that the payment of the premium is done in continuous time, the computation of the mark-to-market leads to the following equivalence.

$$MtM(S_T) = 0 \Longleftrightarrow S_T = \frac{(1-R)\displaystyle\int_0^T e^{-rt} d\mathbb{P}(\tau \le t)}{\displaystyle\int_0^T e^{-rt}\mathbb{P}(\tau > t)dt} \tag{2}$$

[3]If, at initiation of the contract, the contractual spread $S^c_{T_k} \neq S_{T_k}$, then, the mark-to-market is not zero so that one counterparty must pay something to the other counterparty.

[4]See however Beumee et al (2009) for a discussion.

where S_T is the market spread (at the current time $t = 0$) for a CDS of maturity T, and MtM denotes the mark-to-market. For simplicity, we assume that the risk-free rate r is flat.

2.2 Using the market spread of CDS to imply the parameters

To estimate the parameters of the model, it is usual to calibrate (or extract) the parameter(s) of the probabilistic model from the market quotes. Once the model is calibrated, it is used to price exotic products. In the credit risk literature, default is generally modeled as the first time a stochastic process hits some threshold. As we shall see, the stochastic variable and the threshold may, or may not have, an economic (or financial) content.

2.2.1 Deterministic intensity models

First suggested in quantitative finance by Jarrow and Turnbull (1995), and further developed by Duffie and Singleton (1999), *intensity models* are constructed to avoid the modeling problem of the actual payoff of bondholders in a default situation (e.g., the fact that the absolute priority rule is often violated...), and are designed for *relative valuation* purpose, i.e., the term structure of the credit spreads is supposed to be given. An intensity model is not intended to give an economic (or financial) explanation of the default process nor of the resulting recovery rate for bondholders.

Let N_t be an inhomogeneous Poisson process. Starting with $N_0 = 0$, for a fixed $t > 0$, N_t is a Poisson random variable with parameter Λ_t, that is:

$$\mathbb{P}(N_t = n) = \frac{e^{-\Lambda_t}(\Lambda_t)^n}{n!} \qquad n \in \mathbb{N} \tag{3}$$

$$\Lambda_t = \int_0^t \lambda_s ds \tag{4}$$

where λ_s is called an *intensity*, which is assumed to be a deterministic function. An intensity model share the two following characteristics.

- The default event is declared when N_t jumps for the first time, i.e.,

$$\tau = \inf\{t > 0 : N_t = 1\} \tag{5}$$

- The recovery rate of bondholders is *exogenous*. For example, it is assumed to be a fraction of the nominal, e.g., 40%.

From equation (3), seen from $t = 0$, since $\mathbb{P}(N_t = 0) = \mathbb{P}(\tau > t) = e^{-\Lambda_t}$, the *default probability* is equal to:

$$\mathbb{P}(\tau \leq t) = 1 - e^{-\Lambda_t} \tag{6}$$

but Λ_t is unknown. To compute this default probability say at time $t = 0$, one must thus calibrate (i.e., extract) Λ_t from market data. Since only few maturities are available, one must simplify the model. The usual way to do this is to assume that Λ_t is a *piecewise step function*:

$$\Lambda_{T_K} = \sum_{k=1}^{K} \lambda_k (T_k - T_{k-1}) \tag{7}$$

where $\lambda_t = \lambda_k$, $\forall t \in]T_{k-1}; T_k]$, i.e., the intensity is constant between two maturities. The computation of the mark-to-market reduces now to a system of K equations with K unknowns $(\lambda_1, ...\lambda_K)$.

$$\begin{aligned} MtM(S_{T_1}, \lambda_1) &= 0 && (8)\\ MtM(S_{T_2}, \lambda_1, \lambda_2) &= 0 && (9)\\ &\vdots \\ MtM(S_{T_K}, \lambda_1, ...\lambda_K) &= 0 && (10) \end{aligned}$$

One first solve equation (8) to obtain $\hat{\lambda}_1$. Then, by reinjecting $\hat{\lambda}_1$ in equation (9), one can get $\hat{\lambda}_2$ and so on... One can now compute the term structure of default probability

$$\hat{\mathbb{P}}(\tau \leq t) = 1 - e^{-\hat{\Lambda}_t} \qquad t \leq T_K \tag{11}$$

which is *implied* by the term structure of the market spreads.

Let us show how one can easily obtain $\hat{\lambda}_1$. Using equation (7), with $k = 1$ and $T_0 = 0$, we obtain that $\Lambda_{T_1} = \lambda_1$ so that, from equation (6), $\mathbb{P}(\tau \leq 1) = 1 - e^{-\lambda_1}$ and $d\mathbb{P}(\tau \leq 1) = \lambda_1 e^{-\lambda_1} dt$. Using now equation (2), it thus follows that $S_{T_1} = (1 - R)\lambda_1$. Since the market spread S_{T_1} is observed and R is the estimated recovery rate (e.g., 40%), one immediatly get that $\hat{\lambda}_1 = S_{T_1}/(1 - R)$. The *implied default probability* at a horizon $h \leq 1$ is just equal to

$$\hat{\mathbb{P}}(\tau \leq h) = 1 - e^{-\left(\frac{S_{T_1}}{1-R}\right)h} \tag{12}$$

Let us point out, once again, that this way of estimating the default probability does not require to know and/or to understand the underlying activity of the firm. While the mark-to-market practice is natural, it clearly gives an incentive to use a simple black-box (i.e., intensity models) and thus to disregard the corporate data of the firm[5]

2.2.2 Structural models

Pioneered by Merton (1974), and further developed by Black and Cox (1976), *structural models* are intended to give an economic (or financial) explanation of the actual default process, but also of the resulting recovery rate. In principle, one can both value debt and equity. The essential variable in these models is the *market value of the assets*, denoted V. It is generally assumed that V_t follows a geometric brownian motion

$$\frac{dV_t}{V_t} = \mu_t dt + \sigma_t dW_t \tag{13}$$

where μ_t and σ_t are the instantaneous drift and volatility, and dW_t is the increment of a standard brownian motion. A structural model share the two following characteristics.

- The default event is declared when V_t falls below some threshold H_t, i.e.,
$$\tau = \inf\{t > 0 : V_t \leq H_t\} \tag{14}$$
- The recovery rate of bondholders in a default situation, is *endogenous* and equal to $\gamma \frac{V_\tau}{N}$, where $\gamma \in [0,1]$ is a parameter that reflects bankruptcy costs. Note that in general $\frac{V_\tau}{N} < 1$

In this approach, bondholders are assumed to perfectly monitor the value of the assets, and are contractually allowed (e.g., via a bond covenant) to declare the default when the value of the assets becomes lower than a threshold. In general, this threshold is related to the nominal N. For example, in their seminal paper, assuming no bankruptcy costs, Black and Cox (1976) consider the following threshold:

$$H_t = Ne^{-\zeta(T-t)} \qquad \zeta > r \tag{15}$$

[5]If a large part of the market participants estimate the default probability with such an intensity model, and take positions according to this default probability, one may wonder if there is any fundamental information in the market price. This reminds me the (well-known) Grossman-Stiglitz paradox. If all the investors try to extract the relevant information from the market price rather than to pay something to obtain it (e.g., to construct and estimate a fundamental model), then, the market price does not contain any information!

where r is the risk-free rate. This means that when the *solvency* of the firm defined as V_t/N is too low, default is declared[6]. While V_t and H_t may (in principle) be estimated using accounting data (e.g., with historical balance sheet informations), in applied works, V_t and H_t are generally treated as non observables latent variables to be calibrated on market data. Let us give some examples.

One of the most popular model used to estimate for example the default probability is the the Merton (1974) model (or a variant). In this simplest structural model, σ_t is constant, the debt is a zero-coupon bond with finite maturity T and nominal N, and default is declared, *at maturity only*, if the value of the assets is lower than the nominal, i.e., if $V_T < N$. To compute the default probability, one thus need to know V_0, the volatility σ, and the nominal N. For example, in Crosbie and Bohn 2003 (see also Bharath and Shumway 2008, Vassoulou and Xing 2004), the nominal is estimated on accounting data and is equal to the short-term debt plus half of the long-term debt. However, V and σ are calibrated on *equity market data* and are thus not estimated on accounting data. In the Merton (1974) model, the value of equity is a simple call option on V whose value $f(V, \sigma)$ is given by the Black-Scholes formula. One way to calibrate V and σ is as follows. Let S_{t_i} be the observed value of equity at time t_i, for $i = 1, 2....n$. Make a first guess $\hat{\sigma}_1$ for the initial volatility. Given $\hat{\sigma}_1$, find V_{t_i} such that $f(V_{t_i}, \hat{\sigma}_1) = S_{t_i}$ for $i = 1, 2...n$. Compute now the volatility $\hat{\sigma}_2$ that results from the V_{t_i}. Let ϵ be a precision parameter, e.g., 10^{-5}. The process stops at (the smallest) step j such that $\hat{\sigma}_{j+1} \in [\hat{\sigma}_j - \epsilon, \hat{\sigma}_j + \epsilon]$, that is, when it has found a set of output $\{V_{t_i}\}_{i=1}^n$ *consistent* with the volatility used in input. Note interestingly that this calibration process does not use the CDS quotes.

In Brigo and Morini (2006), they consider a default threshold which is time dependent and chosen to obtain analytical formula for the default probability, which is in turn used to compute the mark-to-market of the CDS. In their model, the default threshold which is considered is equal to:

$$H_t = H \exp\left(\int_0^t (\mu_s - \sigma_s^2) ds\right) \tag{16}$$

It turns out that the default probability $\mathbb{P}(\tau \leq t)$ is a function of V_0/H

[6]Note that if $\zeta = r$, then, there is some default risk but there is no loss in a default situation since at default time $\tau < T$, $V_\tau = H_\tau$, so that $e^{r(T-\tau)} H_\tau = N$. As a consequence, the spread must be zero. Note that this critically depends of the assumed dynamics in which almost all the sample paths of the stochastic process are continuous functions of time. This would not be true for say a jump-diffusion model.

(and not on V_0 and H separately), and of $\int_0^t \sigma_s ds$. The way they calibrate the model is interesting since they combine equity and CDS market data. They choose a piecewise constant volatility and calibrate the first year volatility (i.e., $\sigma_t = \sigma_1$, $t \in [0, 1]$) from equity data and use H as a fitting parameter (V_0 is set to one). The remaining term structure of the volatility is calibrated with CDS market spreads, and they use a procedure similar to the system of equations (8), (9), and (10).

It thus follows from our discussion that structural models are essentially used as intensity models since the economic or financial variables are not estimated from corporate data but rather implied from market one. One of the reason is the mark-to-market practice, as we said earlier, but also because these models do not provide a satisfactory explanation of the default process. They fail to recognize that default arises in general from a "liquidity" crisis and not from a "solvency" crisis. To take a simple example, a levered firm that generates an operating income much higher than its financial expenses won't default because the value of its assets (e.g., financial, plant and equipment...) fall. One must thus make a clear distinction between liquidity and solvency.

3 Corporate finance foundations for credit derivatives valuation

3.1 Valuation uncertainty

Following the classical fair value hierarchy, Crouhy, Jarrow and Turnbull (2008) distinguish three levels for classifying the type of "fair" valuation employed.

- Level 1. Clear market price
- Level 2. Valuation using prices of related instruments
- Level 3. Prices cannot be observed and model prices need to be used.

Level 1 is concerned with standard credit products such as CDS for which there is a market spread. Level 2 is concerned with credit products that are not quoted in the market but for which prices can be easily derived from the quoted one. Consider for example a CDS with a nonstandard maturity, e.g., 8Y, and assume that an inhomogeneous Poisson process (with deterministic intensity) is used to compute the mark-to-market. Given $\hat{\Lambda}_t$, for $t \in [0, 10]$, one can compute the mark-to-market of such a CDS and find the fair spread S_{8Y} such that $MtM(S_{8Y}, \hat{\Lambda}_{8Y}) = 0$. Level 3 is concerned with illiquid and/or nonstandard instruments, which arises for example if one want to compute the spread of a CDS of some debt issuer for whom there are no available market data. As a consequence, if we want to determine the "fair" spread, the only way is to use a model prices. The valuation of exotic credit derivatives generally falls under level 3. Actually, most quantitative analysts involved in credit derivatives markets are only working for a "good" probabilistic model, i.e., which allows a *simple*, *robust* and *fast* calibration[7] using vanilla quotes. As a consequence, it is not really necessary to have a good knowledge and/or a good understanding of the corporate data of the firm.

We argue that to properly value a credit derivative, whether it is vanilla or exotic, in addition to the probabilistic model, a "fundamental" model should also be developed. By fundamental, we mean a model for which one can estimate the various parameters *independently* of the market data, that is using only corporate data (the kind of models that are supposed to be used and estimated by rating agencies....). Developing fundamental models devoted to be estimated and back tested on corporate data would probably avoid the very complex (credit) products, but this would (perhaps) help to keep some market liquidity during a turmoil period. When markets are no more liquid, the "best" calibration model is of no help to derive the price of an exotic product since its price is by essence only *relative*. Much of the academic effort have been devoted to models designed for calibration (e.g., intensity models, copulas...) to price the increasingly complex credit derivatives products, but very little has been done to construct fundamental models.

3.2 Models from quantitative corporate finance: liquidity and solvency

There is a huge literature in quantitative corporate finance (e.g., structural models) that aims to determine the value of debt and equity of a given firm, which may in turn be used to value credit derivatives. With credit derivatives in view, *liquidity* and *solvency* of the firm are the two critical quantities to be incorporated in a model since they determine respectively the *default probability* and the *recovery rate*.

[7] Robust means that the parameters of the model are stable with respect to a small perturbation of market data, and fast means that one can obtain the result in few seconds, which explains the popularity of analytical formula.

Liquidity refers to the ability of the firm to pay its current financial expenses using the current operating income (EBIT or EBITDA) but also, if necessary, by using the cash-holding (i.e., corporate liquidities on some bank account). Solvency refers to its ability to redeem the principal if the assets were to be sold now. Roughly speaking, liquidity informs about the "distance-to-default", then used to compute (given some model) a *default probability*, while solvency informs about the *recovery rate*, i.e., the ratio defined as the minimum between one and the value of the corporate assets (e.g., tangible) over the principal of the bond[8]. Since the default probability and the recovery rate are the two key ingredients to compute the market-to-market of a credit derivative, e.g., a CDS, they should thus be modeled.

Let X_t be the operating income (e.g., EBIT or EBITDA) of the firm at time t, and V_t be the market value of the assets (tangible and intangible). Actually, X_t and V_t come from the underlying *investment policy* of the firm[9]. Assuming a fixed investment policy, let (C, N, T, α_t) be the *financial policy* of the firm where C is the interests expenses (e.g., coupon) of the debt, N is the nominal, T is the maturity of the debt, and $\alpha_t \in [0,1]$ is the *dividend policy* at time t. Given a financial policy, the net income of the firm at a given time t is equal to $(X_t - C)(1 - T_{ax})$ when $X_t > C$, (T_{ax} is the corporate income tax), and to $(X_t - C)$ in the opposite case[10]. A fraction α_t of this net income, when positive, is distributed as dividends to shareholders and the complimentary fraction $1 - \alpha_t$ goes on some bank account.

In "classical" structural models, (e.g., Black and Cox 1976, Leland 1994), no clear distinction is made between liquidity and solvency. In general, V_t is a simple multiple of X_t, i.e., $V_t = mX_t$, and no cash reserve is possible, i.e., it is implicitly assumed that $\alpha_t = 1$ for all $t > 0$. As a consequence, a levered firm may default at the first time t for which $X_t < C$. However, shareholders may decide to reinject cash to finance $C - X_t$, for example by diluting the existing shares. In classical model such as Leland (1994), shareholders can dilute at no cost while in practice, there are obviously dilution costs (e.g., administrative fees...) so that the decision to maintain or not the firm alive clearly depends of these dilution costs. More generally, in a default payment situation, if they wish to maintain alive the firm, shareholders will consider all the possible solutions (e.g., dilution, chapter 11, bank refinancing, asset

[8]Bondholders can not claim more than the principal of the bond, at least for bond issued at par.

[9]The EBIT can be observed in the income statement while the value of the assets can be observed in the balance sheet. See e.g., Penman (2010) for a good textbook.

[10]The net income is not subject to corporate taxation when $X_t < C$

sale...) and will (probably) choose the cheapest one. If no solution is possible, the firm will then be liquidated. Let us now present a (selected, necessarily biased) review of some recent models that make a clear distinction between liquidity and solvency.

In Braouezec and Lehalle (2008), they consider a discrete time-finite maturity model in which $\frac{\Delta X_t}{X_t}$ is binomially distributed so that X_t always remains positive. At each time t, a fraction $(1-\alpha)$ of the net income (when positive) is placed on cash reserve (i.e., a bank account) and capitalizes at the risk-free rate. Let $e^{r\,\delta t}B_{t-\delta t}$ be this cash reserve at the beginning of time t. When $X_t < C$, if the cash reserve is high enough, shareholders use it to pay the difference $C - X_t$ to bondholders. Otherwise, it defaults. As a consequence, the default threshold is given by $H_t = X_t + e^{r\,\delta t}B_{t-\delta t}$ so that default occurs at the first time for which $H_t < C$. Note that the default threshold is *stochastic* and function of the dividend policy, but, more importantly, it is *observable* using corporate data only, i.e., calibration from market data is not necessary. When default occurs at time t, it is assumed that the firm is instantaneously liquidated so that the bondholders' recovery is equal to $\min\{X_t + e^{r\,\delta t}B_{t-\delta t} + V_t; N + C\}$. This model assumes thus that dilution costs are prohibitively high, indeed infinite, but also that restructuration (e.g., chapter 11) is not possible. These two quite strong assumptions are relaxed respectively in Decamps-Mariotti-Rochet-Villeneuve (2008), and in Broadie et al (2007).

In Decamps-Mariotti-Rochet-Villeneuve (2008), they consider a continuous time-infinite maturity model in which X_t follows an arithmetic brownian motion so that X_t may be negative. In their model, while the firm is unlevered, dilution costs are assumed to be positive but not necessarily infinite, and, as in the previous model, what is not distributed to shareholders as dividends is placed on cash reserve. However, they assume that the amount placed on cash reserve capitalizes at a rate lower than the risk-free rate due to the presence of "agency costs". Roughly speaking, these agency costs measure the managerial mis-use of cash. Let B_t be the cash reserve. In their model, cessation of operating activity[11] arises at the first time for which $B_t < 0$. Of course, the evolution of B_t depends on dilution and agency costs so that the optimal level of cash reserve is an optimal trade-off between these two types of costs. Since the firm is not levered, chapter 11 makes no senses.

In Broadie et al (2007), as opposed to the two previous models, they explicitly consider a continuous time model (indeed solved numerically) in

[11] Strictly speaking, this is not a default situation because the firm is not levered.

which the firm has the possibility to fill under chapter 11. They thus make a clear distinction between the default and the liquidation threshold. Various interesting comparative static results are presented.

While all these theoretical models develop some aspects of the (complex) default process that are not incorporated in "classical" structural models, as far as we know, they all remain to be tested empirically.

4 Conclusion

We have reviewed in this paper some credit risk models that are used in practice to compute say a default probability. We have suggested that in addition to models devoted to calibration, one should also develop quantitative models in which all the parameters can be estimated on corporate data. This would in turn allow to compute the fair spread say of a CDS.

During few years, securitization has been done in a very complex and highly non transparent way. As noted in a somehow provocative paper, Crouhy and Fleuriet (2009) note that many specialists of mortgage backed securities (MBS) and CDO of asset backed securities (ABS) have simply forget the basics of corporate finance, so that many CDO's tranches have been mis-valued (see also Crouhy et al 2008 or Danielson 2008). As far as we know, a fundamental multivariate model that allows to determine the fair spread of a CDO tranche does not exist.

References

[1] Bharath S, Shumway T, (2008), "Forecasting Default with the Merton Distance to Default Model", *Review of Financial Studies*, May, pp. 1339-1369.

[2] Beumee J, Brigo D, Schiemert D, Stoyle G, (2009), "Charting a Course Through the CDS Big Bang", Fitch Solutions, Global Special Report, Quantitative Research.

[3] Black, Cox (1976), "Valuing Corporate Securities : Some Effects of Bond Indenture Provisions", *Journal of Finance*, 31, pp 351-367.

[4] Braouezec Y, Brun J, (2007), *Dérivés de crédit vanille et exotiques*, La Revue Banque.

[5] Braouezec Y, Lehalle C, (2008), "Corporate Liquidity, Dividend Policy and Default Risk: Optimal Policy and Agency Costs", *International Journal of Theoretical and Applied Finance*, forthcoming.

[6] Brigo D, Morini M, (2006), "Structural Credit Calibration", *Risk Magazine*, April.

[7] Broadie M, Chernov M, Sundaresan S, (2007), "Optimal Debt and Equity Values in the Presence of Chapter 7 and Chapter 11", *Journal of Finance*, june, pp 1341-1377.

[8] Crosbie P, Bohn J, (2003), "Modeling Default Risk", Moody's KMV Company

[9] Crouhy M, Fleuriet M, (2009), "La crise des crédits *subprime* et la théorie de la pizza, *Revue Banque*, N^o 710, février, 2009.

[10] Crouhy M, Jarrow R, Turnbull S, (2008), "The Subprime Credit Crisis 2007", *Journal of Derivatives*, Fall, pp 81-110

[11] Danielson J, (2008), "Blame the models", *Journal of Financial Stability*, vol 4, issue 4, pp 321-328.

[12] Das S, Hanouna P, Sarin A, (2008), "Accounting-Based versus Market-Based Cross-Sectional Models of CDS Spreads", mimeo.

[13] Decamps JP, Mariotti T, Rochet JC, Villeneuve S, (2008), "Free Cash-Flow, Issuance Costs and Stock Price Volatility", mimeo.

[14] Duffie D, Singleton K, (1999), "Modeling Term Structure of Defaultable Bonds", *Review of Financial Studies*, vol 12, Num 4, pp 687-762.

[15] Felsenheimer J, Gisdakis P, Zaiser M, (2006), *Active credit Portfolio Management*, Wiley.

[16] Jarrow R, Turnbull S, (1995), "Pricing Derivatives on Financial Securities Subject to Credit Risk", *Journal of Finance* ,50, 1, pp 53-86.

[17] Leland, H, E, (1994), "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, vol. 49(4), pp 1213-52.

[18] Penman S, (2010), *Financial Statement Analysis and Security Valuation*, fourth edition, McGraw-Hill.

[19] Vassouslou M, Xing Y, (2004), "Default Risk in Equity Returns", *Journal of Finance* ,Num 2, April, pp 831-868.