. . . About Hedging

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This column describes how to control risk through the use of financial futures and forward contracts. I begin with a discussion of the valuation of futures and forward contracts. Then I describe how they can be used to change the asset mix of a portfolio without disrupting the underlying assets. Next, I show how to hedge away the systematic risk and extract the alpha from an actively managed portfolio of short and long positions. I also describe how to remove the currency risk of an internationally diversified portfolio, and demonstrate why full hedging is not necessarily optimal, even independent of any consideration about hedging costs. Finally, I show how to evaluate the tradeoff between the cost of hedging and risk reduction.[1]

Valuation of Financial Futures and Forward Contracts

A financial futures contract obligates a seller to pay the value of the futures contract to the buyer at a specified date. Financial futures contracts have uniform terms with respect to quantity, expiration date and underlying asset. Forward contracts, by contrast, are negotiated privately; their terms are thus specific to the transaction.

The fair value of a futures or forward contract is based on the notion of arbitrage. Suppose that the S&P 500 is currently valued at $450.00, that the three-month riskless interest rate is 1.50%, and that the S&P's expected dividend yield for the next three months equals 1.00%. The price of a futures contract on the S&P 500 that expires three months from now should equal $452.25. At this price, we are indifferent between purchasing the S&P 500 on margin or purchasing a futures contract on the index.

Assume that the value of a unit of an S&P 500 index fund equals 500 times the price of the index, and that we purchase a unit with borrowed funds. Now suppose that, after three months, the S&P's price rises to $460, at which time we sell our unit. We receive $232,250—the price for which we sell our unit ($230,000) plus dividends equal to $2,250. At the same time, we must pay $228,375—the principal of our loan ($225,000) plus interest of $3,375, for a net gain of $3,875.

If we instead purchase a futures contract on the S&P 500 priced at $452.25 and sell it at expiration when its price equals $460, we earn the same profit—$3,875. The value of an S&P 500 futures contract equals the contract price times 500. We thus purchase the contract for $226,125 and sell it for $230,000.

What happens if the S&P 500 declines to $440 after three months? In this case, the strategy of purchasing the S&P 500 on margin loses $6,125. We experience a capital loss of $5,000, receive dividend income of $2,250, and incur an interest expense of $3,375. If we purchase a futures contract for $226,125 and sell it for 220,000, we experience the same loss of $6,125.

Table 1 illustrates the equivalence of a futures contract and a leveraged exposure to the underlying asset. In general the value of a futures or forward contract equals the price of the underlying asset plus the cost of carry, which for financial assets is defined as the interest cost associated with purchasing the asset on margin less any income the asset generates during the term of the contract.

Arbitragers monitor the prices of futures contracts and their underlying assets and engage in arbitrage transactions whenever opportunities exist. This activity prevents futures prices from moving significantly away from their fair values. The range of values around fair value is determined by the ease with which arbitragers can profit from a misvalued futures contract. The more expensive or uncertain it is to transact in the underlying asset, the further away from fair value the futures price is likely to drift before arbitragers enter the market.

Table 1. Equivalence of Futures Contract and Leveraged Exposure

	S&P Leveraged	S&P Futures Contract
Purchase Price	225,000	226,125
	(450 × 500)	(452.25 × 500)
Interest Cost	3,375	0
	(0.015 × 225,000)	
Dividend Income	2,250	0
	(0.01 × 225,000)	
Sale Price	230,000	230,000
	(460 × 500)	(460 × 500)
Profit/Loss	3,875	3,875
Sale Price	220,000	220,000
	(440 × 500)	(440 × 500)
Profit/Loss	−6,125	−6,125

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Asset Allocation with Futures Contracts

Suppose we have a $100 million portfolio, 60% of which is allocated to an S&P 500 index fund and 40% of which is allocated to a 20-year Treasury bond with a coupon yield of 8%. Also, suppose we wish to reduce its stock exposure to 40% and increase its bond exposure to 60%. One approach would be to sell the underlying S&P securities and invest the proceeds in Treasury bonds. Alternatively, we can sell S&P futures contracts and buy Treasury bond futures contracts as an overlay to our portfolio.

Because the value of an S&P 500 contract equals 500 times its price, we determine the number of contracts to sell as follows. We divide the value of the position we wish to trade ($20,000,000) by the quantity 500 times the S&P index price. At an S&P price of $450, we should sell 89 S&P futures contracts [20,000,000/(500 × 450.00)].[2]

Suppose that a 20-year Treasury bond with an 8% coupon is currently priced at $110-16/32. The value of a Treasury bond contract equals 1000 times the price of this reference bond. We should therefore purchase 181 contracts in order to increase our bond exposure proportionately [20,000,000/(1000 × 110.50)].

Now consider a situation in which we have a $100 million S&P 500 index fund, and we wish to convert 20% of it to Treasury bills. As was true in the previous example, we need to sell 89 S&P 500 futures contracts in order to reduce our stock exposure, but we need not purchase any Treasury bill futures contracts. Recall from the discussion about valuation that a futures contract is priced so that it is equivalent to purchasing the underlying asset on margin. The price of the contract equals the price of the underlying asset plus the implicit interest cost less the forgone dividend income. Therefore, by selling a futures contract against the underlying asset, we in effect create a Treasury bill exposure.

To see this equivalence, again suppose that the S&P 500 is valued at $450, that its dividend yield over the next three months equals 1.00%, and that the three-month riskless yield equals 1.50%. As shown earlier, a futures contract on the S&P 500 will be priced at $452.25.

Now suppose, as we did earlier, that the S&P price three months from now rises to $460. The $20 million exposure to the S&P index yields dividend income equal to $200,000 and a capital gain of $444,444.44. At the same time, the 89 S&P futures contracts that were sold produce a capital loss of $344,875, for a net gain of $299,569.44, which is equivalent to a 1.5% yield. Had we been able to sell fractional contracts, this arbitrage would have generated precisely $300,000.

If the price of the S&P 500 falls to $440 three months hence, the $20 million exposure to the S&P index fund generates a capital loss of $444,444.44 and dividend income of $200,000. The short futures position produces a capital gain of $545,125, for a net gain of $300,680.56, which again equals a yield of 1.5%.

Whether the S&P index rises or falls, a long position in the index together with an offsetting short exposure to S&P 500 futures contracts yields the riskless return, as long as the futures contracts are priced fairly. If they are priced below their fair value, this strategy will generate a return below the Treasury bill yield. If they are overpriced relative to their fair value, this strategy will generate a premium over the Treasury bill yield.

Now consider a situation in which we wish to change the allocation of a portfolio that consists of an actively managed equity component and an actively managed bond component. We do not wish to vitiate the value we expect to add through active management. We wish only to reallocate the assets so as to reduce our portfolio's exposure to the systematic risk of its equity component and to increase its exposure to the systematic risk of its bond component.

We measure the systematic risk of our equity portfolio by regressing its returns on the returns of the market, as represented by the S&P 500. The slope of the regression line is called beta, and it represents the sensitivity of our fund's return to the market's return. For example if our fund's beta equals 1.2, and the S&P index returns 10%, we should expect our fund to return 12%. The extent to which its return is above or below 12% can be attributed to the active management of the fund.[3]

We compute the number of S&P contracts to sell in order to account for our equity portfolio's systematic risk as shown in Equation 1:

$$N = A/S \times \beta \tag{1}$$

where

N = number of contracts to trade,
A = amount to be reallocated,
S = 500 times S&P 500 index price and
β = beta of equity component.

If our equity component has a beta of 1.2 and we wish to reduce our portfolio's systematic equity risk 20%, we should sell 107 S&P 500 futures contracts. Although the value of these contracts is greater than $24 million, this transaction acts to lower our portfolio's systematic equity risk by only 20% while retaining its full exposure to security selection skill.

If we wish to increase our portfolio's bond exposure by 20%, we must determine the number of Treasury bond futures contracts that matches the systematic risk of our portfolio's bond component.

A bond's systematic risk is measured by its sensitivity to changes in the level of interest rates. This measure is called duration, and it equals the average time to receipt of a bond's cash flows weighted by their present values. If a bond's duration equals 10 and interest rates decline by one percentage point, the price of the bond will increase by 10%. Duration differs from term to maturity in two ways. First, term to maturity measures the time to receipt of the final principal repayment, whereas duration measures the average time remaining to receipt of all the cash flows, including coupon payments and interim principal repayments.

Second, duration is weighted by the present values of the cash flows.[4]

If we wish to increase our portfolio's systematic bond risk by an amount equal to 20% of its value, we need to adjust the number of Treasury bond futures contracts that we acquire, as shown in Equation 2:

$$N = A/S \times D_P/D_T. \quad (2)$$

where

N = number of contracts,
A = amount to be reallocated,
S = 1000 times underlying Treasury bond price,
D_P = duration of portfolio bond component and
D_T = duration of Treasury bond that underlies futures contract.

Suppose that the duration of the Treasury bond that underlies the futures contract equals 12, while the duration of our bond component equals 10. If, as assumed earlier, the value of a 20-year Treasury bond with an 8% coupon yield equals $110,500, we should purchase 151 contracts.[5] Although this transaction increases our bond exposure by only $16.61 million, it has the effect of creating a $20 million additional exposure to a bond with a duration equal to 10.

Short/Long Strategies

Suppose we wish to focus on stock selection and immunize our portfolio from broad market movements. That is, we wish to purchase stocks that we believe will outperform the market and to sell short stocks that we believe will underperform the market. If our long position and our short position have the same beta, we can eliminate systematic risk by purchasing and selling equal amounts. If, however, our long and short positions have different betas, then we must adjust our portfolio's long and short exposures to account for the difference in their betas if we want to eliminate systematic risk.

Suppose, for example, that our long position has a beta equal to 1.0 while the beta of our short position equals 0.9. We can eliminate systematic risk in two ways. We can limit our long position to 90% of our short position. This approach, however, places more emphasis on our ability to identify stocks with negative alphas than to identify stocks with positive alphas. As an alternative, we can establish equal long and short exposures to the individual stocks and sell S&P 500 futures contracts to offset 10% of our long position. This approach reduces the systematic risk of our long position to that of our short position, while maintaining equal exposure to the stock-specific risk of our long and short positions.

If we believe that our stock selection skill is limited to identifying stocks that we expect to outperform the S&P 500, we can neutralize our market exposure by selling S&P futures contracts in an amount based upon the beta of our long position. If we feel more comfortable identifying stocks that we expect to underperform the market, we can eliminate our fund's market exposure by purchasing S&P contracts in an amount based upon the beta of our short position.

Hedging Currency Exposure

The notion of beta can also be applied to hedging currency exposure. Suppose we allocate a fraction of our portfolio to overseas investments. How much of the embedded currency risk of these investments should we hedge? One approach is to sell currency forward or futures contracts in an amount equal to the currency exposure of our investments. Typically, however, this approach will not minimize the currency risk of our portfolio. We are more likely to reduce currency risk if we condition the amount we hedge on the beta of our portfolio with respect to the relevant currency.

Suppose that 30% of our portfolio is allocated to the Japanese stock market. Assume that our portfolio has a standard deviation of 12%, that the yen has a standard deviation of 10%, that our portfolio is 15% correlated with the yen, and that independent of any change in the value of the yen, our portfolio has an expected return of 10%. We can think of this independent return as our portfolio's alpha with respect to movements in the yen. In order to minimize the volatility of our portfolio's return that is associated with changes in the dollar/yen exchange rate, we should sell a forward contract on the yen in amount equal to 18% of our portfolio's value; which is to say, we should hedge 60% of our portfolio's yen exposure.

The reason that we should hedge only 60% of our currency exposure in order to minimize risk is that our portfolio's beta with respect to the yen is 18%, and 18% of our 30% yen exposure equals 60%.

A portfolio's beta with respect to the yen equals its correlation with the yen times its standard deviation divided by the yen's standard deviation, as Equation 3 shows:

$$\beta = \rho \times \sigma_P/\sigma_C \quad (3)$$

where

β = portfolio beta with respect to currency,
ρ = correlation between portfolio and currency,
σ_P = portfolio standard deviation and
σ_C = currency standard deviation.

In order to determine the effectiveness of this currency hedging strategy, let us assume that the yen will either increase or decrease by 10%. If it increases 10% and we do not hedge any of the portfolio's currency exposure, we should expect the portfolio to return 11.8%—the sum of the 10% expected return that is independent of the yen's return plus 0.18 times the yen's return. By the same reasoning, we should expect the portfolio to return 8.2% should the yen decline by 10%.

Now suppose we hedge all the portfolio's exposure to the yen; that is, we sell a forward contract on the yen equal to 30% of the portfolio's value. If the yen increases by 10%, we should expect the portfolio to return 8.8%—

the sum of the underlying portfolio's return plus the return of the short forward position (−3.0%). If the yen falls by 10%, we add the 3% return from the short forward position to the return of the underlying portfolio; our expected return is now 11.2%.

If instead we sell short a forward contract on the yen in an amount equal to 18% of our portfolio, which equals 60% of our yen exposure, we should expect to eliminate fully the portfolio risk that arises from uncertainty in the yen exchange rate. If the yen rises 10%, the forward position loses 1.8% for a net return of 10.0%. If the yen falls 10%, we gain 1.8% on the short forward position, which when added to the underlying portfolio's return again equals 10%. Table 2 summarizes these results.

Table 2. Effectiveness of Alternative Currency Hedging Strategies

Portfolio Beta with Respect to Yen: 18%
Portfolio Alpha with Respect to Yen: 10%
Portfolio Exposure to Yen: 30%

Alpha +	Beta ×	Yen Return +	Forward Exposure ×	Yen Return =	Portfolio Return
			Unhedged		
10%	18%	10%	0%	10%	11.8%
10%	18%	−10%	0%	−10%	8.2%
			Fully Hedged		
10%	18%	10%	−30%	10%	8.8%
10%	18%	−10%	−30%	−10%	11.2%
			Optimally Hedged		
10%	18%	10%	−18%	10%	10.0%
10%	18%	−10%	−18%	−10%	10.0%

We can also verify that a beta-derived currency hedge ratio minimizes portfolio risk by computing the standard deviation of a portfolio combined with a short position in a currency forward contract, as shown in Equation 4:

$$\sigma_{P+F} = (\sigma_P^2 + \sigma_F^2 \times W^2 + 2 \times \rho \times \sigma_P \times \sigma_F \times W)^{1/2} \quad (4)$$

where

σ_{P+F} = standard deviation of combination of portfolio and forward contract,
σ_P = standard deviation of underlying portfolio,
σ_F = standard deviation of currency forward contract,
W = weighting of currency forward contract, and
ρ = correlation of underlying portfolio and currency forward contract.

By substituting the assumptions given earlier into Equation 4, we find that the standard deviation of the fully hedged strategy equals 11.92%, versus 11.86% for the beta-derived hedging strategy. Although this difference may seem rather small, to the extent we incur transaction costs to hedge currency exposure, the beta-derived hedging strategy is less expensive to implement as long as the beta is less than our portfolio's currency exposure.

Risk Reduction versus Cost of Hedging

In the currency hedging example, I assumed that we wish to minimize portfolio risk as a function of currency exposure regardless of the expected cost of hedging. It is more likely that we would seek to balance risk reduction with the cost of hedging.

The cost of hedging currency risk has several components. There are transaction costs as well as management and administrative fees. In addition, the currency futures or forward contract will sell at discount to the spot exchange rate when domestic interest rates are lower than foreign interest rates. If the spot exchange rate does not decline to the current forward rate, we will incur a loss on our short forward position. The opposite may also be true; we might experience a gain if we sell a currency forward contract at a premium and the spot rate fails to appreciate to the forward rate prevailing at the time we sell the contract.

In any event, to the extent we have reason to believe that a currency forward contract's expected return is different from zero, we should reflect this view in our estimate of the cost of hedging. If we anticipate a positive return, we should raise our estimate of the hedging cost by this amount. If we expect a negative return, we should lower our cost estimate.

Once we estimate the cost of hedging, we need to determine how many units of cost we are willing to incur at the margin in order to lower our portfolio's variance by one unit. We can infer this tradeoff from our choice of the underlying portfolio. It is the slope of a line that is tangent to the efficient frontier at the location of our portfolio's expected return and risk, assuming risk is measured in units of variance.

The exposure to a currency forward contract that optimally balances our aversion to risk with our reluctance to incur costs is given by Equation 5, assuming our portfolio is exposed to only one foreign currency:[6]

$$W = C/(2 \times \lambda \times \sigma_F^2) - \rho \times \sigma_P/\sigma_F \quad (5)$$

where

W = optimal exposure to currency forward contract as a fraction of portfolio value,
C = expected cost of hedging, including expected return of forward contract,
λ = tradeoff between risk reduction and cost,
σ_F = standard deviation of currency forward contract,
σ_P = standard deviation of underlying portfolio, and
ρ = correlation of currency forward contract and underlying portfolio.

Suppose, for example, we estimate hedging costs to equal 0.25% and we determine that we are willing to incur two units of incremental cost to reduce our portfolio's variance by one unit. Based on our earlier assumptions about correlation and standard deviations, we should sell a forward contract on the yen equal to 11.75% of our portfolio's value, which corresponds to 39.17% of its exposure to the yen. Although this hedging strategy does not minimize the risk due to currency exposure, it optimally balances our willingness to incur cost in order to lower risk.

I have attempted to present some of the basic principles of hedging with financial futures and forward contracts. I have ignored much of the administrative detail associated with the application of these principles. Those who are interested in these important details should consult other sources, including those referenced in the notes.

Footnotes

1. Those who are interested in a more detailed review of these topics should see S. Figlewski in collaboration with K. John and J. Merrick, *Hedging with Financial Futures for Institutional Investors: From Theory to Practice* (Cambridge, MA: Ballinger Publishing Company, 1986).
2. This result ignores the fact that financial futures contracts are marked to market daily. As a consequence, gains and losses accrue in an interest-bearing margin account. In order to adjust for this daily mark-to-market feature, we should reduce the hedge position to its present value. This adjustment is called "tailing the hedge."
3. According to the Capital Asset Pricing Model, we should estimate systematic risk by regressing a portfolio's returns in excess of the riskless return on the market's excess returns. To the extent the riskless return is stable, though, this approach will yield a similar estimate.
4. For a more detailed description of duration, see M. Kritzman, "What Practitioners Need to Know about Duration and Convexity," *Financial Analysts Journal*, November/December 1992.
5. Treasury bond futures contracts can be settled by delivery of a variety of bonds. Therefore, it is sometimes necessary to adjust the hedge ratio by a delivery factor that equates a deliverable bond to the reference bond.
6. This framework assumes that we only sell currency forward contracts. If we were to consider purchasing forward contracts, we would reflect the hedging cost net of a forward contract's expected return as a negative value.